Tail-risk of bulls and bears:

The tale of developed and emerging markets

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Abstract

The accuracy of estimates of Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) hinge on whether the assumed distributional form of returns is correctly specified. Such a correct specification, however, is a major issue, as there is no consensus on what distributional form provides the best fit with the empirical data. Indeed, many researchers argue that the best fit is obtained when mixtures of distributions are used. In this research, we study whether and how the distributional properties of stock market returns differ for bull and bear markets, and how these differences impact on the accuracy of the estimates of the tail-risk measures. We show that bull and bear markets have different distributional forms for developed and emerging stock exchanges. These differences have important implications for the accuracy of the methods used to estimate VaR and CVaR.

JEL classification: G15, G17, G10, G19

Keywords: return distributions, tail risk, Value-at-Risk, Conditional-Value-at-Risk, developed stock exchanges, emerging stock exchanges, bull markets, bear markets

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Introduction

Mark Zuckerberg, the CEO and co-founder of Facebook, said that "the biggest risk is not taking any risk". However, when it comes to taking risk it is important to know how much risk one takes, so it is important to be able to quantify risk. While the standard deviation is a fundamental concept of theoretical asset pricing and portfolio theory (Treynor, 1961, 1962; Sharpe, 1964; Lintner, 1965), financial regulators and practitioners have been working closely with Value-at-Risk (VaR) since the early 1920s when it was used to specify capital requirements for firms listed on the New York Stock Exchange (Holton, 2002). The rapid development of international capital markets, growth of institutional investors, and regulatory changes following the 2007 financial crisis, contributed greatly to the popularisation of VaR and Conditional Value-at-Risk (CVaR) as an important tool in asset management and regulation. Yet, in spite of numerous papers and books being written on the conceptualisation of the tail-risk measures, and how to calculate them in practice (e.g., Alexander, 2008), it is still unclear what distributional forms best capture tail shapes, how these distributions change with market conditions, and how accounting for these changes improves VaR and CVaR estimates.

Using the correct distributional form of returns is crucial for accuracy of tail-risk measures calculations, but finding these correct distributional forms is not an easy task. It seems that mixing distributions gives a better fit than using a single form specification, but still it is unclear what distributions and in what proportions should be mixed (Ball and Torous, 1983; Kon, 1984; Peiró, 1994; Press, 1967; Merton, 1976; Kim and Kon, 1994). This paper adds to the literature by fitting distributional forms to returns of bull and bear markets in a sample of 23 stock exchanges (13 developed and 10 emerging) over the period of January 2000 – December 2016. It shows that there are distributional differences between bull and bear markets, and between developed and emerging stock exchanges. It also shows that utilising the knowledge of the best distributional fit informs on the suitability of using historical data and the Extreme Value Theory (EVT) to estimate VaR and CVaR.

The computations of VaR and CVaR requires a long time series of data, if one uses historical data to estimate the tail-risk, or the knowledge of the distribution of the underlying processes. However, in real life, very often neither long time series are readily available nor the form of the distribution is known. Moreover, to complicate the story, it is broadly understood that distributional moments change with changes in market conditions. So-called bull markets are associated with positive mean returns, while bear markets are associated with negative means and higher standard deviations than those observed for bull markets. This is, at least partly, a consequence of the fact that investors behave differently in bull and bear markets, and that investors' behaviour is endogenous to market conditions (Gervais and Odean, 2001; Chalmers *et al*., 2013).

It is well documented that there are considerable differences in distributional characteristics of equity returns across stock exchanges (e.g., Harris and Küçüközmen, 2001; Bris *et al*. 2007; Theodossiou and Savva, 2015) and, in particular, that these differences are strongly pronounced for developed and emerging stock markets (e.g., Bekaert and Harvey, 1997; Bekaert *et al.*, 1998; Morck *et al*., 2000; Aggarwal *et al*., 2009). It is, however, unclear whether and how distributional properties change with bull and bear market conditions. It can be expected that markets of higher risk have higher probability of so-called three-sigma events, hence, the distributions of the emerging stock exchanges' returns may have thicket tails than the distributions of the returns of the developed stock exchanges. However, while this observation may be true 'on average', it may not hold in all market conditions. For instance, when markets fall, the decline in prices may be equally dramatic on the developed and the emerging stock exchanges. Therefore, bear markets, whether they are experienced by the developed or the emerging stock exchanges, may have similar statistical properties. Bull markets, on the other hand, are typically much shorter on the emerging stock exchanges than they are on the developed stock exchanges. Hence, one could argue that if there are differences between distributional properties of the developed and of the emerging stock exchanges, these might be more pronounced during bull markets rather than during the bear markets.

To assess distributional characteristics of the sample, the normal distribution, Student's t distribution (Student, 1908), Hansen's Skewed t distribution (Hansen, 1994), and Skewed Generalised t distribution (Theodossiou, 1998) are used. For each stock exchange, these distributions are fitted to the 16 years of daily returns (the whole period) and to its sub-periods of bull and of the bear markets. Lunde and Timmermann's (2004) method is adopted to separate individual time series into bull and bear markets. The –

Log Likelihood test, the Akaike Information Criterion and the Schwartz Bayesian Criterion are employed to assess the quality of the distributional fit. Next, VaR and CVaR at 99% confidence intervals are calculated using these distributional specifications as well as using historical simulations and the EVT.

We find that indeed, there are different distributional specifications for bull and bear markets. We also find that it is the developed exchanges that are more prone to changing their distributional form than the emerging stock exchanges between the bull and bear markets. The comparison of the estimates of VaR and CVaR shows that, as expected, historical simulations deliver poor proxies for the 'true' tail risk. Moreover, in contrast with the previous literature, we also find that the EVT based estimations are prone to considerable errors. They seem to overestimate VaR and underestimate CVaR.

The paper, therefore, contributes to the literature in four significant ways. First, it improves our understanding of the distributional forms of daily stock market returns. Second, it improves our understanding of the quality of different methods of estimation of VaR and CVaR. Third, it casts new light on the differences between distributional forms of stock returns of the developed and of the emerging stock exchanges and, finally, it shows differences between bull and bear markets.

The rest of the paper is organised as follows. Section 2 provides a brief literature review and introduction to the main research questions behind this research. Section 3 defines the sample, describes the methodology used to separate the period of investigation into the bull and the bear markets and provides specifications of the VaR and CVaR estimation methods. Section 4 discusses the results of empirical fitting of distributions, and Section 5 discusses the differences across estimated VaRs and CVaRs. Section 6 closes with conclusions.

1. Literature review

If there is a consensus on the distributional from of equity returns, it is that there is no universal distribution nor a universal family of distributions which fit the time series of returns. In the early days, the Gaussian distribution was proposed (Bachelier, 1900; Osborne, 1959; Moore, 1962; Akgiray and Booth, 1987). However, these days it is well recognised that it is extremely rare to observe normally distributed time series of equity returns, especially at daily frequency. Several alternative distributions have been used to better fit the data. These include: Paretian distribution (Mandelbrot, 1967; Fama, 1963; Fama, 1965; Officer, 1972; Clark, 1973), the power exponential distribution (Nelson; 1991; Hsieh, 1989; Theodossiou, 1994; Koutmos and Theodossiou; 1994; Akgiray and Booth, 1988), Student's t distribution (Praetz, 1972; Blattberg and Gonedes, 1974; Praetz, 1972; Kon, 1984; McDonald and Newey, 1988; Gray and French, 1990; Peiró, 1994; Aparicio and Estrada, 2001; Kim and Kon, 1994), Skewed Student’s t (HSt) distribution (Hansen, 1994), and Skewed Generalized t distribution (Theodossiou, 1998; McDonald and Nelson, 1989; Butler, McDonald et al., 1990; Lye and Martin; 1993; McDonald and Xu, 1995; Harris and Küçüközmen, 2001), to names the most popular ones.

In addition, many studies claim that using a mixture of distributions, rather than a single distribution, offers a better fit with empirical data as distributional mixes are better at modelling distributions with non-zero skewness and excess kurtosis (Ball and Torous, 1983; Kon, 1984; Peiró, 1994; Press, 1967; Merton, 1976; Kim and Kon, 1994). To add to the difficulty of finding the best distributional form, distributional moments are not time-invariant. For instance, volatility clustering is well recognised and modelled in the finance literature. [2] From the perspective of this research, it is important that changes in distributional moments are not random. For instance, periods of low and of high volatility are associated with specific market conditions such as markets of growth and of decline in prices.

It is well-established in the finance literature that even if the returns are unpredictable, hence the assumption of market efficiency is not broken, markets have periods of upwards and downward trends. These periods of upward and downward trends are often referred to as bull and bear markets, respectively. While the existence of bull and bear markets is widely observed (Lucas Jr, 1978; Ball, Cecchetti *et al*., 1990; Basu and Vinod, 1994; Siegel and Coxe, 2002; Keynes, 1937; Galbraith, 1979; Shiller, 1992; Allen and Gorton, 1993; Allen, Morris *et al*., 1993), considerably less attention

[2] For instance, GARCH specifications have been used to model time-varying second moments (e.g., Engle, 1982; Bollerslev, 1986). In addition, to the negative skewness (Black, 1976; Blanchard and Watson, 1982; Christie, 1982; Pindyck, 1984; Poterba and Summers, 1986; French, Schwert et al., 1987; Schwert, 1989; Campbell and Hentschel, 1992; Bekaert and Wu, 2000), the time varying conditional skewness was investigated (Chen, Hong et al., 2001).

is paid to distributional forms of returns during these markets. However, as it can be expected that distributional moments change between bull and bear markets (e.g., positive means and low standard deviations are associated with bull markets while negative means and high standard deviations are associated with bear markets). It may also be true that distributional families change when markets' conditions change. If it is the case, such distributional changes will have a strong effect on the shape of tails and, therefore, the accuracy of estimates of the tail-risk measures.

The identification of bull and bear markets have attracted considerable attention. There is no precise definition of a bear and of a bull market, although it is commonly understood that a bull market is a market of rising prices and low (below average) volatility, while a bear market is characterised by declining prices and high (above average) volatility. Given the flexibility in defining how much lower/higher volatility should be to be categorised as an indication of a bull/bear market, and how to identify turning points of rising/declining prices, numerous approaches have been developed to separate markets into bull and bear periods. Broadly speaking, the methods of bull/bear market identification can be divided into non-parametric and parametric ones. Non-parametric techniques are based on set rules that tie returns and volatility (e.g., Bry and Boschan, 1971; Lunde and Timmermann, 2004; Pagan and Sossounov, 2003).

In terms of the parametric models, Markov regime switching model developed by Hamilton (1989) can be regarded as a parameterised algorithm for computing turning points. Markov-switching model has been used extensively to capture the cyclical patterns of asset prices (Schaller and Norden, 1997; Hamilton and Lin (1996); Gordon and St-Amour, 2000) and to investigate the duration-dependence issues of the equity markets (Maheu and McCurdy, 2000a, 2000b).[3]

In this research, Lunde and Timmermann's (2004) method is adopted. It relies on finding turning points of market states by using sample observations to decide the values of a series of binary variables indicating the particular market state. Specifically, it is assumed that the stock market switches from a bull/bear state to a bear/bull state if stock prices have declined/increased by a certain percentage since their previous local peak/trough within the bull/bear state. This method does not rule out sequences of

[3] Macroeconomics variables have also been used to identify bull and bear markets. For instance, Chen (2009) finds that term spreads and inflation rates are useful predictors of recessions (bear market) in the US stock market.

negative/positive price movements in stock prices during bull/bear markets as long as the cumulative value caused by the sequence of changes does not surpass the default threshold. Then, a sequence of binary variables is constructed: valued as unity between troughs and peaks indicating bull markets and zero between peaks and troughs indicating bear markets. This method focuses on changes in asset values and ignores the durations of different phases assuming that capital changes are most relevant concerns for investors. The observed durations of different phases have no clear pattern. For the purpose of this research it is important that Lunde and Timmermann's (2004) method does not use distributional moments to separate bull and bear markets. In this way, it sidesteps the problem of endogeneity in defining market states.

Another strand of research that is fundamental to this study concerns with methods of estimation of tail risk. Over the last two decades numerous methods have been proposed to calculate tale risk. Each of these methods has some caveats. For instance, historical simulations are commonly criticised for their implicit assumption that the distributions of returns are time-invariant, since they can be expected to differ across market regimes. Second, they are consistent only if the sample size goes to infinity, which is often unrealistic. Third, VaR estimates obtained by historical simulations are likely to have predictable jumps because of the discreteness of extreme returns. So, even though they are not affected by potential model misspecification, they are subject to other potential drawbacks.

On the other hand, EVT estimates, introduced to financial applications by Koedijk *et al.* (1990) and Jansen and De Vries (1991), are good at identifying asymmetries and fat tails. However, once again, the underlying assumption of i.i.d. is inconsistent with the characteristics of financial returns[4]. In addition, EVT works only for very low probability levels because we need to estimate the parameters based on extreme observations. However, the choice of the threshold to obtain extreme values is potentially an issue.

In this research, we address the issue of the suitability of historical simulations and EVT by comparison of their VaRs with those obtained for a range of distributional specifications. As we expect that the separation of the observations into the bull and the

[4] Although generalizations to dependent observations have been proposed (e.g., Leadbetter *et al*., 2012; Embrechts *et al.*, 2013), they either estimate the marginal unconditional distribution or impose conditions that rule out the volatility clustering behaviour typical of financial data.

bear markets may magnify asymmetries and fatness of the tails, the accuracy of the methods will be tested in different states of the market.

2. Data and methodology

3.1. Sample description

For the purpose of empirical testing, a daily stock price movement of indexes of 13 developed stock exchanges (DSEs) and 10 emerging stock markets (ESEs) have been collected from DataSteam. The sample covers the period of 01 January 2000 – 31 December 2016. The quarterly price indexes for the same sample are collected from Q1 2000 to Q4 2016[5]. The sample and period were selected to (i) maximise the sample size and coverage of the important changes that took place on the world stock exchanges, i.e., their collapse after the burst of the dotcom bubble, and after the credit crunch and collapse of the subprime mortgage market, (ii) ensure that bull and bear periods are long enough to provide reliable estimates of distributional parameters. The length of the individual time series differs because each stock exchange has a different number of traded days (e.g., nontraded days have been removed). For each time series daily returns for traded days are calculated.

The DSEs in the sample come from Canada, Finland, France, Germany, Greece, Italy, The Netherlands (NL), New Zealand, Portugal, Singapore, Sweden, Switzerland, the United Kingdom (UK) and the United States of America (US). The ESEs are from Argentina, Chile, China, Columbia, India, Peru, Philippines, Romania, Saudi Arabia, Taiwan, and Turkey. Therefore, the sample covers a wide range of stock markets at different levels of development, and operating on all continents.

The basic sample statistics are presented in Table 1 for the DSEs and in Table 2 for the ESEs. In addition to individual markets' statistics, the averages for each group of stock markets are provided. It is clear that based on the group averages, consistent with common wisdom, the ESEs seem more volatile (their standard deviation is larger) than that of the DSEs. They are also having larger negative skewness and larger kurtosis suggesting that, the distribution of daily returns of the average ESE has thicker tails and,

[5] The quarterly data is used to separate the market into bull period and bear periods. All the calculations and simulations are conducted on daily data.

it has the left tail thicker than the average DSE. Normality of the distributions of daily returns is rejected for every DSE and ESE.

*************** insert Table 1 here ************

*************** insert Table 2 here ************

However, a closer look at the statistics of individual countries shows that both groups of SEs are highly heterogeneous in the sense that some ESEs are less volatile and thinner tailed than some of the DSEs. For instance, the standard deviation of the Chilean SE is lower than the standard deviation of every DSE. The Canadian SE has the skewness larger than any ESE, and the kurtosis of the Taiwanese SE is smaller than kurtosis of every single DSE.

3.2. Separation into bull and bear markets

Using the Lunde and Timmermann’s (2004) method with the (-15%, 15%) threshold on quarterly data we separate each times series of returns into the bull and the bear periods. Hence, each SE has its own timing and duration of the bull and the bear markets. For each SE the observations from its bull (bear) periods are pooled together to create a bull (bear) sample which will be referred to as bull (bear) market. These bull(bear) markets will be used to determine the statistical and distributional properties of the bull and the bear periods for each SE. Table 3 Panel A is analogous to Table 1 except that it shows the number of observations and summary statistics for the bull markets for the DSEs. Table 3 Panel B repeats the exercise for the DSE’s bear markets. Table 4 Panel A and Panel B are analogous to Table 3 Panel A and Panel B but present the corresponding statistics for the ESEs.

*************** insert Table 3 here ************

*************** insert Table 4 here ************

Tables 3 and 4 show that, on average, the DSE's and the ESE's bull and bear markets have similar numbers of observations. On average, the bear markets of the DSEs account for 33% of the observations while those of the ESEs for 34% of the observations. There are, however, considerable differences across countries. In the DSE sample Greece is the country with the largest number of bear market observations (49%), while Canada, the UK and the US have the lowest proportion of bear market observations (about 25%). In the sample of the ESEs, China has the highest proportion of bear market observations (52%) while Turkey and India have the lowest (26%).

Table 3 and Table 4 show that, as expected, the standard deviation is higher for the bear markets than for the bull markets for both the DSEs and the ESEs. Only in the case of Columbia is the standard deviation of the bull market slightly lower than the standard deviation of the bear market. More dynamics are observed in the case of skewness and kurtosis. These moments change quite significantly between the bull and the bear markets, and in particular, for the ESEs. Given that Lunde and Timmermann's (2004) method allows for negative (positive) corrections during bull (bear) markets, both the bull and the bear markets may have 'outliers' in the sense that during the periods of market rises (declines), short periods of negative (positive) returns (so-called corrections) can occur. Such switches may be more visible for the ESEs than for the DSEs causing quite dramatic impact on distributional properties.

To illustrate this, Figures 1 and 2 plot the daily movement of the S&P500 index for the US and the BIST National 100 index for Turkey respectively (blue lines). They also show how the period of the analysis is divided into the bull and the bear periods (orange lines). The high levels of market state line correspond to the bull periods, while the opposite is true for the bear periods.

*************** insert Figure 1 here ************

*************** insert Figure 2 here ************

The US index is one of the indexes with the lowest number of switches, only three bull periods and two bear periods. In contrast, the Turkish index has the highest number of switches with seven bull markets and six bear periods.[6] It is clear that Lunde and Timmermann's (2004) method detects the up and the down trends quite accurately, but in the case of the Turkish index which, in aggregate, is twice as volatile as the US index (Table 1 shows that US standard deviation of daily returns is 1.24% and Table 2 shows that the corresponding Turkish standard deviation is 2.18%) there is more volatility within individual bull and bear periods. This increased volatility causes potentially contra intuitive changes in skewness and kurtosis between the bull and the bear markets. The Turkish bull periods have thicker, in aggregate, tails and are more skewed to the left than the bear periods. A similar observation can be drawn for several ESEs, especially those more volatile ones with many switches between the bull and the bear states.

To better visualise these differences, Figures 3 and 4 show kernel densities for the US and the Turkish indexes respectively. Panels A and Panels B show the kernel densities for the bull and the bear markets respectively. It is clear that the bull markets' distributions have smaller standard deviations and smaller tails (thinner and shorter) than the bear markets. This is particularly visible for the US bull market. The asymmetry of the tails is also clear. The kernel densities obtained for the Turkish index confirm thick tails in the bull market as indicated by kurtosis of 11 and more weight in the left tail during the bear market as indicated by skewness of -0.312 (Table 4).

****************** Figure 3 *****************

****************** Figure 4 *****************

3.3. Distributional fit and estimation of VaR and CVaR

Value-at-Risk, VaR, of a random variable x at the confidence level α is the value of the quantile of the inverse function of its cumulative distribution function (CDF) at the probability level α, i.e., $\mathrm{VaR} = F^{-1}(\alpha)$, where F denotes the CDF of x and α denotes the probability such that $F(\mathrm{X}) = \mathrm{P}(x < \mathrm{X}) = \alpha$. In other words, $F(\mathrm{VaR}) =$

[6] Finland also has a high number of switches with six bull and seven bear periods, but we discuss Turkey in more detail as an example of a high-volatility emerging stock exchange.

$\mathrm{P(x < VaR) = \alpha}$. Conditional-Value-at-Risk (CVAR), is the expectation of all the values of *x* below the threshold level α, i.e.,

$$CVaR_{\alpha} = E[x|x \leq \mathrm{VaR}_{\alpha}] = \int_{-\infty}^{VaR_{\alpha}(x)} xf(x)dx$$

While the definitions of VaR and CVaR are straightforward, finding their numerical values is less so. The definitions of VaR and CVaR are based on the clear specification of the distributional properties of the underlying time series. As discussed in Section 2 there is no consensus on what distributional specifications time series of stock market returns have. Moreover, our hypothesis is that these distributional properties may change with changes in market conditions. Therefore, to capture possible changes in the first four moments, we consider four widely discussed distributional specifications: normal (N), Student's t (St), Hansen's Skewed-t (HSt) and Skewed Generalised-t (SGt).

Following Theodossiou (1998) SGt is defined by the density function:

$$\mathrm{f(x|\nu, k, \lambda, \sigma)} = \begin{cases} C(1 + (\frac{\mathrm{k}}{\nu - 2})\theta^{-k}(1-\lambda)^{-k}\left|\frac{x}{\sigma}\right|^{k})^{-\frac{(\nu+1)}{k}}, x < 0 \\ C(1 + (\frac{\mathrm{k}}{\nu - 2})\theta^{-k}(1+\lambda)^{-k}\left|\frac{x}{\sigma}\right|^{k})^{-\frac{(\nu+1)}{k}}, x \geq 0 \end{cases}$$

where ν, k, λ, and σ are scaling factor parameters. C and θ are the normalizing scalars ensuring that f(·) is a proper probability density function. For the above probability density function, C and θ are given by:

$$C = \frac{1}{2\sigma} kB\left(\frac{1}{k}, \frac{\nu}{k}\right)^{-\frac{3}{2}} B\left(\frac{3}{k}, \frac{\nu - 2}{k}\right)^{\frac{1}{2}} S(\lambda)$$

$$\theta = \frac{1}{S(\lambda)}\left(\frac{k}{\nu - 2}\right)^{\frac{1}{k}} B\left(\frac{1}{k}, \frac{\nu}{k}\right)^{\frac{1}{2}} B\left(\frac{3}{k}, \frac{\nu - 2}{k}\right)^{-\frac{1}{2}}$$

with S given by:

$$S(\lambda) = \left[1 + 3\lambda^2 - 4\lambda^2 B\left(\frac{2}{k}, \frac{\nu-1}{k}\right)^2 B\left(\frac{1}{k}, \frac{\nu}{k}\right)^{-1} B\left(\frac{3}{k}, \frac{\nu-2}{k}\right)^{-1}\right]^{\frac{1}{2}}.$$

Thanks to the flexibility of SGt, the other three distributions used in the study are nested within the SGt model. That is, the HSt distribution is defined by the imposition of $k = 2$, the St distribution by restricting $\lambda = 0$ and $k = 2$, and the normal distribution by imposing $\lambda = 0$, $k = 2$, and $\nu = \infty$.

For the purpose of this research the parameters of the distributions are estimated by maximizing the log-likelihood of the relevant probability density function. The optimization algorithm of Nelder and Mead (1965) is adopted and the starting values are obtained from Theodossiou (1998).

There are no closed form expressions for the relationship between VaR/CVaR and the parameters of the distributions used in the analysis. Hence, we calculate VaR by numerical integration and the Bisection method. To be more specific, we implement the Bisection method to find the value x which makes cumulative density value (integral of probability density function from $-\infty$ to x) equal to the associated confidence interval α. VaR is then defined as the absolute value of x.

In order to calculate CVaR for a given distribution, we generate 10,000 random numbers for this distribution specification and find the α quantile of the random number sample. Then, the absolute value of the average of all the values smaller than α quantile value is taken. In this research, α is 1%.

The EVT estimates are obtained from the Generalized Pareto Distribution to model the extreme values. The value of the threshold parameter is 5% value of the sample. We use the same parameters for all the EVT calculations.

To assess the quality of the distributional fit the –Log Likelihood test (-LogL), the Akaike Information Criterion (AIC) and the Schwartz Bayesian Criterion (SBC) are calculated. –LogL statistics are provided for the completeness of the analysis as it is expected that the criterion will be biased towards the distribution with the highest number of parameters and may lead to ‘overfitting’. Given that both AIC and SBC ‘penalise’ for the number of parameters, they are of the main interest to us. Given that AIC has several advantages over SBC (e.g., AIC is derived from principles of information and has the prior that is a declining function of the number of parameters), more weight will be put on the AIC than on the SBC outcomes.

4. Empirical distributions of returns

We start the analysis of distribution specifications from fitting each of the four distributions described in Section 3.3 to each time series of daily returns for the whole period and then for the bull and the bear markets. The results for the whole period for the DSEs and the ESEs are presented in Tables 5 and 6 respectively. Tables 7 and 8 show the results for the DSEs for the bull and the bear markets respectively, and Tables 9 and 10 show the results for the ESEs for the bull and the bear markets respectively. Each table has three panels. Panel A shows the estimates of the parameters for each of the four distributions used in the study except for the means which are already presented in Tables 1- 4. Panel B presents diagnostic statistics for -LogL, AIC and SBC for each of the distribution specifications and for each SE. Panel C shows the likelihood ratios (LR) and their corresponding P-values for tests of whether the restrictions imposed on the SGt to reduce its form to the N, St or HSt distributions are valid. The null hypothesis is that the restrictions are valid, i.e., the simplified version of SGt is preferred to SGt itself.

Panels A in Tables 5 and 6 show that while the degrees of freedom, ν, for the St and HSt are comparable, the degrees of freedom calculated for the SGt can be considerably higher. This increase is more pronounced for the ESEs than for the DSEs. The results presented in Table 5 Panels B show that SGt offers the best fit according to the –LogL criterion for every DSE. This is not surprising as the –LogL does not penalise for the increase in the number of parameters used to identify the distributions. The AIC and the SBC, which correct for the increase in the number of parameters defining the distributions, challenge the –LogL. According to AIC, for 10 out of 13 DSEs the SGt is the best specifications. However, in the case of Canada and Switzerland, the St distribution is best. For the US, the HSt is best. A stronger departure from the SGt specification is reported for the SBC, for which seven out of 13 DSEs are better described by less parameter intensive distributions, i.e., St or HSt.

Table 5 Panel C shows that the null-hypothesis for testing whether the SGt parameter restrictions are valid in comparison with the N, St and HSt specifications cannot be rejected for every DSE. Therefore, the Likelihood ratio tests do not confirm that SGt offers the best fit.

*************** insert Table 5 here ************

*************** insert Table 6 here ************

The results obtained for the ESEs are more diverse. The lowest AIC in support of SGt is obtained for seven exchanges, for the remaining three (Chile, Columbia and Turkey) St or HSt distributions had the lowest AIC. The normal distribution is not supported for any exchange in the sample. The SBC, as the criterion that sets higher penalties than the AIG for using additional parameters, confirms that SGt is not the optimal specification for Chile, Columbia and Turkey. In addition, the SBC indicates that for the exchanges of India and Peru, it might be better to use the HSt specification.

The Likelihood ratios, once more, offer conflicting results. The Likelihood ratio tests show that the null hypothesis is rejected for Chile and Turkey for the comparison of SGt with St and HSt. However, the AIC and the SBC indicated that HSt was the best fit for Chile and St was best for Turkey.

In summary, if we were to put more weight on the AIC, then we would conclude that using SGt may be necessary to describe the data for the DSEs. Interestingly, in the case of some ESEs simpler distributional specifications may be sufficient.

The separation into the bull and the bear markets offers more consistency across different diagnostic tests. Tables 7 and 8 show results analogous to those presented in Table 5, save for the fact that they show the results for the DSEs for the bull and the bear markets respectively.

*************** insert Table 7 here ************

*************** insert Table 8 here ************

Table 7 shows that according to the AIC and the Likelihood ratio tests, the SGt specification is best for ten out of 13 exchanges. For Greece, Singapore and Sweden, the null hypothesis of validity of restrictions to reduce SGt to St or HSt cannot be rejected. It cannot also be rejected for Sweden, although, the AIC's lowest value is obtained for SGt.

Table 8 shows that in the case of the bear markets the SGt specification may not be necessary. This time, the AIC indicates SGt as the best fit only for Italy. The SBC uniformly supports the AIC. Moreover, the likelihood ratios show that the null hypothesis can be rejected for the normal distribution only. Therefore, there is some evidence that the bull and the bear markets may have different distributional forms for DSEs with the bull markets being more 'complicated' than the bear markets.

Tables 9 and 10 show the results analogous to those presented in Tables 7 and 8 but for the ESEs. Table 9 shows that, according to the AIC, distributions of returns of five ESEs should be modelled with SGt and the other five with less parameter demanding specifications, i.e., St or HSt, during the bull markets. The SBC confirms this finding and additionally strengthens it, indicating SGt as the optimal specification for three ESEs only. The results of AIC are also supported by the likelihood ratio test.

*************** insert Table 9 here ************

*************** insert Table 10 here ************

During the bear markets, the number of the ESEs that require SGt increases to six, the four that required it during the bull markets plus Columbia and India (Argentina is dropped). For five of these markets the likelihood tests reject the null hypothesis, i.e., confirm superiority of the SGt specification. The results of the information criteria and of the likelihood ratio tests suggest that for the remaining ESEs using the St or the HSt specifications would be sufficient.

In summary, the ESEs seem more robust in their distribution form, i.e., exchanges that require SGt during the bull markets are also likely to need SGt during the bear markets, too. In contrast, many DSEs that have their returns best described with SGt during the bull markets would not need SGt during the bear markets. St or HSt specifications would be sufficient.

5. Tail risk

Given that there are considerable differences between which distributions may be best for different stock exchanges during the bull and the bear markets, in this section potential differences in tail risk estimates will be discussed. As indicated in Section 3.3, there are numerous techniques to estimate the tail risk, and there is no consensus which of these methods is best. Given that it can be expected that the size of the tail risk is highly sensitive to whether it is calculated for the bull or the bear markets, and, as Section 4 showed, there are considerable differences in the form and shape of the distributions for these two market states and across the exchanges, the tail risk calculated for the N, St, HSt, SGt distribution specifications. In addition, the tail risk obtained using historical simulations and the EVT will be discussed. All these estimates of tail risk will be compared separately for the bull and the bear markets. First, VaR and then CVaR will be discussed.

5.1 VaR

We start from the comparison of the VaR estimates for the DSEs and the ESEs for the whole period of investigation, i.e., without the separation into the bull and the bear markets. Figure 5 Panel A show the comparison of the VaR estimates for the DSEs and Panel B for the ESEs. Both figures show that VaRs based on historical simulations are lowest. Considerably higher values are obtained for the normal distribution, and higher still for the St, HSt and SGt distributions with VaR obtained for the St distributions being slightly higher than those for the HSt and the SGt distributions. Differences across these three density specifications are relatively small for the majority of the DSEs and ESEs. VaRs based on the EVT are highest amongst of the VaR estimates.

******************** Insert Figure 5 here **************

Given that for nearly all DSEs and ESEs the preferred distribution is SGt (according to the AIC), and the SGt's VaRs are typically smaller than the VaR estimated for St and HSt, it is more likely that the SGt's VaRs are more accurate.

Therefore, the EVT VaRs may be considerably overstated. The differences between the EVT VaRs and SGt VaRs vary between 0.08% for Finland and 0.81% for Saudi Arabia. On average, this difference is 0.22% for the DSEs and 0.37% for the ESEs. Separating the observations between the bull and the bear markets casts new light on this.

Figure 6 shows the estimates of the VaR for the DSEs for the bull (Panel A) and the bear (Panel B) markets respectively.[7] As previously, both figures show that VaRs based on historical simulations are lowest. Considerably higher values are obtained for the normal distribution, and higher still for the St, then the HSt and the SGt distributions. The largest VaRs are, once more, obtained for the estimates based on the EVT.

First, as discussed in the case of the VaR estimates for the entire sample we can reject the estimates based on the historical simulations and the assumption of the normality of returns because the normal distribution specification was uniformly rejected for the daily returns for the DSEs during the bull markets by –LogL, AIC, BSC and LR tests. Therefore, given that the estimates of VaR for the normal distribution are lower than any of the other VaR estimates obtained for St, HSt and SGt, it can be concluded that they are too low. This is consistent with the observation that distributions of returns have tails thicker than the normal distribution would indicate. Similarly, given that VaR estimated from historical simulations are much lower than VaR estimated under the assumption of the normal distribution of returns, we can conclude that these historical VaRs are grossly underestimated, and can be rejected.

*************** insert Figure 6 here ************

There seem to be small differences across the VaRs obtained for the three variants of the ‘t-family’ distribution. One pattern can be observed: VaR of St is not smaller than the VaR of HSt, which in turn, is not smaller than the VaR of SGt is common across the 13 SEs in the sample. Given that the SGt distribution was preferred by the AIC for the vast majority of the DSEs, and it is associated with the lowest values of VaR, the VaRs estimated by the EVT seem too large. On average, the difference

[7] The numerical data behind these and the other Tables presented in this Section are presented in Appendix.

between EVT VaRs and SGt VaRs is 0.35% with the biggest difference of 0.58% observed for the Greek stock exchange.

Therefore, these results confirm that in the case of the DSEs the VaRs estimated from the historical data and from the EVT seems least accurate.

Figure 6 Panel B shows that, consistent with our intuition, VaR increases during the bear markets. It also confirms that the VaR estimates based on historical simulations and the N distribution are far below those obtained for St, HSt and SGt. Interestingly, this time the pattern of the St estimates being no less than those of HSt that are no less than SGt that has been observed for the whole period and the bull markets does not hold. For all the exchanges the HSt estimates of VaR are smallest among the St, HSt and SGt specifications. The HSt specification was also the preferred one according to the AIC criterion for many markets. The differences across the estimates of the 't-family' are small and may seem practically negligible, but those lowest ones are, in most cases, noticeably smaller than the EVT estimates. The avegage difference between the EVT VaRs and the HSt VaRs is 0.20% with the highest difference of 0.41% observed for the US exchange. This, once more, suggests that the EVT estimates may exaggerate the true values of VaR.

Figure 7 shows the results of VaR estimates for the ESEs. Again, Panel A shows the results for the bull markets while Panel B shows the estimates for the bear markets. Figure 7 shows that there is a large variety across ESEs. While the VaR estimates for Chile are smaller than any estimates for the DSEs, Argentina's and Turkey's are much larger. The pattern observed for the DSEs holds for the ESEs, as well, i.e., the historical and the N distribution based estimates of VaR are considerably smaller and the EVT estimates are considerably larger than those for the t-family. As with the results obtained for the DSEs, we observe that the smallest values of the t-family correspond to the preferred distributional specifications according to the AIC.

*************** insert Figure 7 here ************

Figure 7 Panel B shows that the bear markets' VaRs are larger than VaRs estimated for the bull markets except for Columbia. However, the increase in the VaR estimates is visibly smaller than it was for the DSEs. All the estimates of the EVT VaRs are considerably larger than the estimates of VaR for the preferred distribution from the t-family except for the SE of Taiwan.

Therefore, we can conclude that, for the DSEs and ESEs, the VaR estimates based on historical simulations, the assumption of the normal distribution of returns or EVT seem biased.

5.2 CVaR

A similar conclusion can be drawn for the estimates of CVaR. Figures 8 and 9 show the estimates for the DSEs and the EMEs respectively.[8] As previously, Panels A show the estimates for the bull markets and Panels B show the estimates for the bear markets.

*************** insert Figure 8 here ************

*************** insert Figure 9 here ************

Once more, the historical and the N distribution based estimates are much lower than any other estimates regardless of the type of the exchanges (developed or emerging) and specifics of the markets (bull or bear). The story with the EVT estimates is slightly more complicated this time. In the case of the DSEs during the bull markets, the EVT estimates tend to be no larger than the t-family based estimates but comparable with the estimates for the SGt specifications where these were preferred by the AIC (Figure 8 Panel A). In other words, we do not have a potential overvaluation observed for the VaRs.

However, in the case of the bear markets, the EVT estimates tend to be considerably smaller than the estimates obtained for the HSt and the St specifications which are preferred based on the AIC (Figure 8 Panel B). Hence, the EVT estimates seem to underestimate the CVaR for the vast proportion of the DSEs.

[8] To save space we do not present the results for the whole period. These results can be obtained from the authors on request. As in the case of the VaR results, the whole sample CVaR results are similar to the CVaR bull market results, as the proportion of the bull observation is higher than the proportion of the bear observations.

Figure 9 confirms some issues with reliability of the EVT estimates both for the bull and the bear markets. For some ESEs in the bull markets (Figure 9 Panel A) the EVT based estimates are lower than the estimates based on the AIC selected distributional forms (e.g., Peru), but for the majority of the exchanges in the sample, they are larger. During the bear markets, on the other hand, the EVT based estimates tend to be consistently too low in comparison with the estimates based on the AIC preferred specifications within the t-family.

In summary, in the case of the CVaR estimates, the EVT estimates as well as these based on the historical data or the assumption that the returns are normally distributed, are poor proxies of the CVaRs obtained for the time series of returns when St, HSt or SGt specifications are used.

6. Conclusions

We investigate distributional forms of daily equity returns for bull and bear markets for 13 developed and 10 emerging stock exchanges over the period 01 January 2000 – 31 December 2016. We use Lunde and Timmerman's (2004) method with the (-15%, 15%) threshold on quarterly data to separate times series of the stock exchanges into bull and bear periods. Each stock exchange has different timing and duration of bull and bear periods. The U.K. and the U.S. have the smallest number of such periods (three bulls and two bears), and Turkey has the highest number of bull-bear periods (seven bulls and six bears). Given that individual bull and bear periods are too short to allow for credible fitting of distributions, for each exchange we pool all its bull periods' observations in one sample which we refer to as the bull market and use these observations to find the best distribution fitting for the exchange. We do the same with the bear periods' observations, and call the pooled sample - the bear market.

We find that in developed markets bull and the bear markets have different distributional forms. Daily returns of 12 out of the 13 developed stock exchanges in the sample require the Student's t or the Hansen's Skewed t distribution during the bear markets. However, during the bull markets, Student's t or Hansen's Skewed t, is sufficient for three of these twelve stock exchanges only. The other exchanges require the Skewed Generalised t distribution.

In the case of the emerging stock exchanges there was less homogeneity within the bull and the bear markets but more consistency across the bull and the bear markets. That is, neither the bull nor the bear markets could be uniformly associated with a particular form of distribution, but those countries where daily returns were best fitted with the Skewed Generalised t distribution during the bull markets, were also best fitted with the Skewed Generalised t distribution during the bear markets. Four exchanges had 'simpler' distributions during the bull markets than during the bear markets, which is the opposite to what we observed for the developed stock exchanges. Only one emerging stock exchange, Argentina, was best fitted with the Skewed Generalised t distribution during the bull markets and the 'simpler' Hansen's Skewed t distribution during the bear markets.

Therefore, our analysis shows that there are substantial differences in both the distributional forms that best fit different stock exchanges during the bull and the bear markets and how these distributional forms change across the two market states. This is consistent with the argument that mixtures of distributions may be more suitable than single distributional forms to describe distributions of daily equity returns.

These results help understand the suitability of historical simulations and EVT based simulations of VaR and CVaR. It is argued that EVT outperforms the other volatility based models, especially for high confidence interval tail-risk measure estimates, because of its ability to model fat tails (e.g., Gencay and Selcuk, 2004; Chan and Gray, 2006).

Our results confirm that historical based simulations underestimate VaR and CVaR. However, the EVT based calculations do not seem to provide more reliable estimates. In comparison with the estimates obtained for the preferred distributional forms (according to the Akaike Criterion) the EVT estimates tend to overestimate VaR but underestimate CVaR for both the developed and emerging stock exchanges.

These results are important for risk managers and regulators using VaR and CVaR to set capital requirements for banks and other financial institutions, and in stress tests. The results highlight the importance of searching for the best distributional fit and understanding the specifics of periods from which observations have been drawn rather than relying on a particular method of estimation. They also show differences between the developed and the emerging stock exchanges, and highlight a high level of heterogeneity across the emerging markets. This calls for more studies on the distributional properties of returns and potential factors driving them.

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Table 1. Descriptive statistics of the daily returns of the stock market indexes of the developed stock exchanges for 1 January 2000 – 31 December 2016. J-B Test denotes the Jaque Bera test and KS-N Test denotes the Kolmogorov–Smirnov Normality Test. P-values of the tests are provided in brackets.

	Canada	Finland	France	Germany	Greece	Italy	NL	Portugal	Singapore	Sweden	Switzerland	UK	US	Average
Sample Size	4,270	4,268	4,345	4,321	3,895	4,314	4,340	3,891	4,264	3,766	4,276	4,295	4,276	4,194
Maximum	0.094	0.146	0.103	0.108	0.134	0.109	0.098	0.113	0.073	0.099	0.108	0.088	0.110	0.106
Minimum	-0.098	-0.172	-0.094	-0.089	-0.175	-0.133	-0.094	-0.104	-0.086	-0.088	-0.091	-0.087	-0.095	-0.108
Mean *100	0.024	0.003	0.011	0.012	-0.029	-0.004	0.007	0.002	0.014	0.029	0.013	0.018	0.018	0.009
Stdev*10	0.113	0.182	0.143	0.154	0.192	0.157	0.137	0.121	0.113	0.146	0.120	0.116	0.124	0.140
Skewness	-0.642	-0.257	-0.072	-0.047	-0.361	-0.194	-0.206	-0.210	-0.368	0.032	-0.169	-0.209	-0.192	-0.223
Kurtosis	12.121	9.455	7.749	7.182	9.493	7.943	9.101	10.070	8.800	7.263	9.683	8.878	11.149	9.145
J-B Test	15,094	7,456	4,087	3,151	6,926	4,419	6,762	8,132	6,073	2,852	7,977	6,214	11,858	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
KS-N Test	0.481	0.471	0.478	0.475	0.472	0.477	0.477	0.482	0.480	0.477	0.479	0.481	0.479	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	

Table 2. Descriptive statistics of the daily returns of the stock market indexes of the emerging stock exchanges for 1 January 2000 – 31 December 2016. J-B Test denotes the Jaque Bera test and KS-N Test denotes the Kolmogorov–Smirnov Normality Test. P-values of the tests are provided in brackets.

	Argentina	Chile	China	Columbia	India	Peru	Romania	Saudi Arabia	Taiwan	Turkey	Average
Sample Size	4,174	4,242	4,112	3,781	4,214	4,249	4,254	4,195	4,200	4,261	4,168
Maximum	0.161	0.091	0.094	0.147	0.150	0.128	0.115	0.164	0.065	0.178	0.129
Minimum	-0.130	-0.060	-0.093	-0.111	-0.129	-0.133	-0.131	-0.117	-0.099	-0.200	-0.120
Mean*100	0.082	0.033	0.019	0.061	0.040	0.050	0.064	0.030	0.002	0.036	0.042
Stdev*10	0.216	0.075	0.164	0.130	0.152	0.140	0.159	0.154	0.141	0.218	0.155
Skewness	-0.166	-0.137	-0.340	-0.167	-0.550	-0.424	-0.402	-0.605	-0.244	-0.066	-0.310
Kurtosis	7.095	13.131	7.491	15.145	10.479	14.526	11.893	15.069	6.215	10.365	11.141
J-B Test	2,936	18,153	3,535	23,256	10,034	23,646	14,132	25,718	1,850	9,633	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000	(0.000)	(0.000)	(0.000)	(0.000	
KS-N Test	0.469	0.487	0.476	0.478	0.476	0.477	0.474	0.475	0.477	0.468	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000	(0.000)	(0.000)	(0.000)	(0.000	

Table 3. Descriptive statistics of the daily returns of the stock market indexes of the developed stock exchanges for the bull markets (Panel A) and the bear markets (Panel B) within the period of 1 January 2000 – 31 December 2016. J-B Test denotes the Jaque Bera test and KS-N Test denotes the Kolmogorov–Smirnov Normality Test. P-values of the tests are provided in brackets.

	Canada	Finland	France	Germany	Greece	Italy	NL	Portugal	Singapore	Sweden	Switzerland	UK	US	Average
Panel A														
Sample Size	3,204	2,960	3,005	2,612	1,984	2,475	3,136	2,176	2,822	2,640	3,148	3,220	3,207	2,739
No of periods	4	6	4	4	4	4	4	4	3	3	4	3	3	
Maximum	0.052	0.086	0.088	0.066	0.096	0.107	0.076	0.113	0.058	0.099	0.057	0.051	0.069	0.078
Minimum	-0.056	-0.090	-0.079	-0.063	-0.144	-0.068	-0.057	-0.055	-0.045	-0.075	-0.091	-0.052	-0.069	-0.073
Mean*100	0.059	0.033	0.056	0.063	0.104	0.057	0.049	0.064	0.080	0.075	0.044	0.046	0.050	0.060
Stdev*10	0.093	0.145	0.120	0.131	0.159	0.134	0.117	0.102	0.097	0.137	0.101	0.098	0.103	0.118
Skewness	-0.378	-0.213	-0.149	-0.180	-0.642	-0.067	-0.166	0.270	0.052	-0.004	-0.512	-0.194	-0.263	-0.188
Kurtosis	6.190	7.212	6.249	5.652	9.548	6.899	6.468	12.110	6.830	8.306	7.939	5.699	7.675	7.444
J-B Test	1435	2211	1333	779	3680	1570	1586	7551	1726	3097	3337	997	2957	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
KS-N Test	0.484	0.477	0.482	0.478	0.476	0.480	0.481	0.484	0.484	0.477	0.484	0.484	0.482	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
Panel B														
Sample Size	1,066	1,308	1,340	1,709	1,911	1,839	1,204	1,715	1,442	1,126	1,128	1,075	1,069	1,454
No of periods	3	7	3	3	4	4	3	5	4	4	3	2	2	
Maximum	0.094	0.146	0.103	0.108	0.134	0.109	0.098	0.097	0.073	0.089	0.108	0.088	0.110	0.104
Minimum	-0.098	-0.172	-0.094	-0.089	-0.175	-0.133	-0.094	-0.104	-0.086	-0.088	-0.081	-0.087	-0.095	-0.107
Mean*100)	-0.080	-0.067	-0.090	-0.066	-0.167	-0.086	-0.103	-0.077	-0.116	-0.080	-0.075	-0.066	-0.078	-0.089
Stdev*10	0.159	0.246	0.183	0.183	0.220	0.182	0.180	0.142	0.137	0.165	0.162	0.158	0.174	0.176
Skewness	-0.578	-0.185	0.078	0.093	-0.147	-0.192	-0.105	-0.342	-0.500	0.141	0.183	-0.093	-0.012	-0.127
Kurtosis	10.294	7.237	6.696	6.814	8.469	7.400	8.098	8.007	8.242	5.684	7.827	7.734	8.872	7.798
J-B Test	2423	986	764	1038	2389	1495	1306	1825	1711	342	1102	1005	1536	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
KS-N Test	0.476	0.467	0.472	0.471	0.468	0.475	0.471	0.480	0.479	0.479	0.474	0.477	0.473	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	

Table 4. Descriptive statistics of the daily returns of the stock market indexes of the emerging stock exchanges for the bull markets (Panel A) and the bear markets (Panel B) within the period of 1 January 2000 – 31 December 2016. J-B Test denotes the Jaque Bera test and KS-N Test denotes the Kolmogorov–Smirnov Normality Test. P-values of the tests are provided in brackets.

	Argentina	Chile	China	Columbia	India	Peru	Romania	Saudi Arabia	Taiwan	Turkey	Average
Panel A											
Sample Size	2,891	3,063	1,987	2,433	3,091	2,244	3,054	3,013	2,653	3,134	2,278
No of periods	5	4	6	5	4	3	4	5	4	7	
Maximum	0.161	0.091	0.087	0.147	0.150	0.083	0.115	0.090	0.065	0.178	0.117
Minimum	-0.130	-0.050	-0.093	-0.111	-0.129	-0.079	-0.131	-0.070	-0.069	-0.200	-0.106
Mean*100	0.163	0.059	0.125	0.117	0.092	0.202	0.126	0.105	0.070	0.087	0.115
Stdev*10	0.215	0.071	0.153	0.138	0.140	0.120	0.149	0.115	0.121	0.213	0.143
Skewness	-0.206	0.174	-0.579	-0.169	-0.531	0.229	-0.290	-0.153	-0.209	-0.094	-0.183
Kurtosis	6.998	15.227	7.621	17.033	14.160	8.181	12.756	11.510	6.804	13.034	11.333
J-B Test	1946	19096	1879	19976	16185	2530	12155	9104	1619	13153	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
KS-N Test	0.470	0.488	0.479	0.477	0.479	0.481	0.475	0.478	0.479	0.468	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
Panel B											
Sample Size	1,283	1,179	2,125	1,348	1,123	2,005	1,200	1,182	1,547	1,127	1,412
No of periods	5	3	5	4	4	3	5	4	5	6	
Maximum	0.126	0.045	0.094	0.051	0.068	0.128	0.101	0.164	0.062	0.121	0.096
Minimum	-0.114	-0.060	-0.089	-0.080	-0.107	-0.133	-0.119	-0.117	-0.099	-0.111	-0.103
Mean*100	-0.100	-0.035	-0.080	-0.039	-0.103	-0.119	-0.094	-0.161	-0.115	-0.106	-0.095
Stdev*10	0.218	0.086	0.172	0.115	0.182	0.157	0.182	0.222	0.169	0.231	0.174
Skewness	-0.075	-0.513	-0.148	-0.271	-0.452	-0.606	-0.466	-0.465	-0.149	0.022	-0.312
Kurtosis	7.419	9.573	7.369	6.699	5.692	15.638	10.014	9.865	5.011	4.971	8.225
J-B Test	1045	2174	1698	785	377	13466	2504	2364	266	182	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
KS-N Test	0.467	0.485	0.474	0.481	0.474	0.474	0.472	0.469	0.477	0.469	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	

Table 5. Distributional fitting for the developed stock exchanges: Panel A – distributional parameters for the Normal (N), Student's t (St), Hansen's Skewed t (HSt) and the Skewed Generalised t (SGt) distributions; Panel B – the results of the diagnostic tests for the – Log Likelihood test (-LogL), the Akaike Information Criterion (AIC) and the Schwartz Bayesian Criterion (SBC); Panel C – the results of the Likelihood ratio tests (LR) for the null hypothesis that the simplified version of SGt is preferred.

		Canada	Finland	France	Germany	Greece	Italy	NL	Portugal	Singapore	Sweden	Switzerlan	UK	US
Panel A														
Normal	σ	0.011	0.018	0.014	0.015	0.019	0.016	0.014	0.012	0.011	0.015	0.012	0.012	0.012
St	ν	3.093	2.859	3.549	3.460	3.187	3.470	2.839	3.443	3.186	3.356	3.233	3.215	2.766
	σ	0.012	0.021	0.015	0.016	0.021	0.016	0.015	0.013	0.012	0.015	0.013	0.012	0.014
HSt	ν	3.064	2.837	3.552	3.479	3.163	3.439	2.818	3.438	3.163	3.330	3.209	3.189	2.766
	λ	0.015	0.013	-0.002	-0.003	0.013	0.015	0.015	0.003	0.015	0.015	0.015	0.015	0.001
	σ	0.012	0.021	0.015	0.016	0.021	0.017	0.016	0.013	0.012	0.016	0.013	0.012	0.014
SGt	ν	3.432	4.963	5.053	7.309	5.238	6.416	3.839	6.379	4.283	4.795	3.765	4.199	5.196
	λ	0.015	0.001	0.015	-0.001	0.001	0.000	0.013	0.016	0.017	0.012	0.018	0.017	0.015
	k	1.794	1.377	1.577	1.339	1.444	1.410	1.552	1.384	1.589	1.568	1.741	1.627	1.284
	σ	0.012	0.018	0.014	0.015	0.019	0.016	0.014	0.012	0.011	0.015	0.012	0.012	0.012
Panel B														
-LogL	Normal	-13066.60	-11044.86	-12298.59	-11915.41	-9866.38	-11811.92	-12449.35	-11640.67	-13074.26	-10580.06	-12837.35	-13057.68	-12687.98
	St	-13601.02	-11498.08	-12630.72	-12233.48	-10232.98	-12137.19	-12956.26	-11970.76	-13485.81	-10876.73	-13284.61	-13463.46	-13226.51
	HSt	-13600.74	-11497.20	-12630.73	-12233.51	-10232.33	-12136.15	-12955.81	-11970.79	-13485.11	-10875.57	-13283.88	-13462.70	-13226.52
	SGt	-13601.66	-11509.74	-12634.30	-12248.40	-10241.56	-12148.78	-12961.44	-11981.91	-13489.51	-10880.20	-13285.17	-13466.12	-13243.02
AIC	Normal	-26129.20	-22085.73	-24593.18	-23826.81	-19728.76	-23619.83	-24894.70	-23277.34	-26144.51	-21156.12	-25670.70	-26111.35	-25371.97
	St	-27196.03	-22990.15	-25255.44	-24460.96	-20459.96	-24268.37	-25906.52	-23935.53	-26965.62	-21747.46	-26563.22	-26920.92	-26447.02
	HSt	-27195.48	-22988.40	-25255.46	-24461.02	-20458.65	-24266.31	-25905.63	-23935.58	-26964.22	-21745.15	-26561.76	-26919.39	-26447.03
	SGt	-27193.32	-23009.48	-25258.60	-24486.81	-20473.11	-24287.56	-25912.87	-23953.82	-26969.02	-21750.40	-26560.34	-26922.24	-26476.03
SBC	Normal	13058.24	11036.50	12290.21	11907.03	9858.11	11803.55	12440.97	11632.40	13065.90	10571.82	12828.99	13049.31	12679.62
	St	13588.48	11485.54	12618.15	12220.92	10220.58	12124.63	12943.70	11958.36	13473.27	10864.38	13272.07	13450.91	13213.97
	HSt	13588.20	11484.66	12618.17	12220.95	10219.92	12123.60	12943.25	11958.39	13472.58	10863.22	13271.34	13450.15	13213.97
	SGT	13580.76	11488.84	12613.36	12227.48	10220.89	12127.86	12940.50	11961.25	13468.62	10859.61	13264.27	13445.21	13222.11
Panel C														
N	LR	1070.12	929.76	671.42	666.00	750.35	673.73	1024.17	682.48	830.51	600.28	895.64	816.89	1110.07
	P value	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
St	LR	1.286	23.329	7.158	29.853	17.152	23.189	10.351	22.293	7.402	6.938	1.123	5.314	33.008
	P value	0.268	1.000	0.933	1.000	0.999	1.000	0.984	1.000	0.940	0.926	0.228	0.850	1.000
HSt	LR	1.833	25.076	7.134	29.788	18.463	25.253	11.244	22.238	8.796	9.249	2.576	6.843	33.001
	P value	0.392	1.000	0.932	1.000	1.000	1.000	0.990	1.000	0.968	0.974	0.538	0.923	1.000

Table 6. Distributional fitting for the emerging stock exchanges: Panel A – distributional parameters for the Normal (N), Student's t (St), Hansen's Skewed t (HSt) and the Skewed Generalised t (SGt) distributions; Panel B – the results of the diagnostic tests for the – Log Likelihood test (-LogL), the Akaike Information Criterion (AIC) and the Schwartz Bayesian Criterion (SBC); Panel C – the results of the Likelihood ratio tests (LR) for the null hypothesis that the simplified version of SGt is preferred.

		Argentina	Chile	China	Columbia	India	Peru	Romania	Saudi Arabia	Taiwan	Turkey
Panel A											
Normal	σ	0.022	0.008	0.016	0.013	0.015	0.014	0.016	0.015	0.014	0.022
St	ν	3.302	3.646	2.910	3.071	3.251	2.560	2.470	2.018	2.999	3.481
	σ	0.023	0.008	0.018	0.014	0.016	0.017	0.020	0.072	0.016	0.022
HSt	ν	3.300	3.626	2.893	3.057	3.226	2.556	2.462	2.076	2.977	3.479
	λ	0.002	0.015	0.013	0.014	0.005	0.013	0.015	0.001	0.014	-0.002
	σ	0.023	0.008	0.019	0.014	0.016	0.017	0.020	0.036	0.016	0.022
SGT	ν	6.233	3.517	7.406	3.618	4.148	3.031	3.589	3.327	16.568	4.046
	λ	0.001	0.015	0.015	0.015	0.005	0.015	0.016	0.014	0.001	0.016
	k	1.384	2.057	1.189	1.708	1.651	1.674	1.428	1.111	1.083	1.766
	σ	0.022	0.008	0.016	0.013	0.015	0.015	0.017	0.017	0.014	0.022
Panel B											
-LogL	Normal	-10076.38	-14712.59	-11076.69	-11044.18	-11650.81	-12118.62	-11578.98	-11568.51	-11947.65	-10257.90
	St	-10400.21	-15139.14	-11472.66	-11560.00	-12066.49	-12850.97	-12261.38	-12658.80	-12264.04	-10663.14
	HSt	-10400.22	-15138.69	-11472.83	-11559.95	-12066.61	-12851.19	-12261.47	-12656.83	-12263.49	-10663.15
	SGt	-10411.24	-15138.75	-11495.92	-11561.79	-12069.75	-12853.64	-12270.59	-12691.85	-12293.10	-10663.00
AIC	Normal	-20148.75	-29421.18	-22149.38	-22084.35	-23297.62	-24233.25	-23153.96	-23133.01	-23891.31	-20511.80
	St	-20794.42	-30272.27	-22939.32	-23114.00	-24126.97	-25695.93	-24516.76	-25311.60	-24522.07	-21320.27
	HSt	-20794.45	-30271.37	-22939.65	-23113.90	-24127.21	-25696.38	-24516.94	-25307.67	-24520.99	-21320.30
	SGt	-20812.48	-30267.49	-22981.84	-23113.57	-24129.51	-25697.27	-24531.18	-25373.69	-24576.19	-21316.01
SBC	Normal	10068.04	14704.24	11068.37	11035.94	11642.47	12110.27	11570.63	11560.17	11939.31	10249.54
	St	10387.71	15126.61	11460.18	11547.64	12053.97	12838.44	12248.85	12646.29	12251.52	10650.60
	HSt	10387.72	15126.16	11460.34	11547.59	12054.09	12838.66	12248.94	12644.32	12250.98	10650.61
	SGT	10390.40	15117.87	11475.12	11541.19	12048.89	12832.75	12249.70	12670.99	12272.24	10642.11
Panel C											
N	LR	669.73	852.31	838.46	1035.22	837.88	1470.02	1383.21	2246.68	690.88	810.21
	P value	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
St	LR	22.056	-0.780	46.516	3.579	6.535	5.337	18.413	66.087	58.120	-0.262
	P value	1.000	0.000	1.000	0.689	0.912	0.851	1.000	1.000	1.000	0.000
HSt	LR	22.031	0.125	46.187	3.671	6.293	4.892	18.238	70.023	59.200	-0.291
	P value	1.000	0.011	1.000	0.701	0.902	0.820	1.000	1.000	1.000	0.000

Table 7. Distributional fitting for the developed stock exchanges for the bull markets: Panel A – distributional parameters for the Normal (N), Student's t (St), Hansen's Skewed t (HSt) and the Skewed Generalised t (SGt) distributions; Panel B – the results of the diagnostic tests for the – Log Likelihood test (-LogL), the Akaike Information Criterion (AIC) and the Schwartz Bayesian Criterion (SBC); Panel C – the results of the Likelihood ratio tests (LR) for the null hypothesis that the simplified version of SGt is preferred.

		Canada	Finland	France	Germany	Greece	Italy	NL	Portugal	Singapore	Sweden	Switzerlan	UK	US
Panel A														
N	σ	0.009	0.014	0.012	0.013	0.016	0.013	0.012	0.010	0.010	0.014	0.010	0.010	0.010
St	ν	3.729	3.222	4.069	3.432	2.997	3.675	3.255	3.358	3.359	3.085	3.935	3.680	3.076
	σ	0.010	0.016	0.012	0.014	0.017	0.014	0.013	0.011	0.010	0.015	0.010	0.010	0.011
HSt	ν	3.682	3.186	4.027	3.400	2.980	3.647	3.225	3.331	3.343	3.063	3.897	3.656	3.055
	λ	0.015	0.015	0.015	0.015	0.013	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
	σ	0.010	0.016	0.012	0.014	0.018	0.014	0.013	0.011	0.010	0.015	0.010	0.010	0.011
SGT	ν	7.056	4.812	8.882	57.625	3.858	7.911	6.646	6.943	4.486	3.697	5.541	6.748	7.118
	λ	0.015	0.012	-0.002	-0.001	0.009	-0.001	0.016	0.015	0.015	0.009	0.015	0.015	0.015
	k	1.424	1.518	1.419	1.065	1.638	1.381	1.353	1.343	1.625	1.709	1.610	1.463	1.249
	σ	0.009	0.015	0.012	0.013	0.016	0.013	0.012	0.010	0.010	0.014	0.010	0.010	0.010
Panel B														
-LogL	Normal	-10426.53	-8339.38	-9020.17	-7620.70	-5399.35	-7158.58	-9510.05	-6895.01	-9067.92	-7588.45	-10002.51	-10339.53	-10137.55
	St	-10620.50	-8585.55	-9170.36	-7775.09	-5598.95	-7310.35	-9738.67	-7078.32	-9287.44	-7855.17	-10206.76	-10519.45	-10425.14
	HSt	-10619.69	-8585.01	-9169.55	-7774.46	-5599.16	-7309.77	-9738.13	-7077.89	-9286.65	-7854.67	-10206.26	-10518.76	-10424.73
	SGt	-10625.75	-8589.43	-9177.23	-7794.36	-5600.46	-7317.62	-9747.03	-7085.69	-9288.98	-7856.25	-10209.11	-10524.33	-10439.08
AIC	Normal	-20849.07	-16674.77	-18036.35	-15237.40	-10794.70	-14313.17	-19016.10	-13786.02	-18131.84	-15172.91	-20001.03	-20675.06	-20271.10
	St	-21235.01	-17165.09	-18334.73	-15544.17	-11191.90	-14614.69	-19471.33	-14150.65	-18568.87	-15704.35	-20407.52	-21032.89	-20844.28
	HSt	-21233.37	-17164.01	-18333.10	-15542.92	-11192.32	-14613.54	-19470.27	-14149.78	-18567.31	-15703.34	-20406.53	-21031.51	-20843.45
	SGt	-21241.50	-17168.87	-18344.46	-15578.72	-11190.92	-14625.23	-19484.05	-14161.38	-18567.96	-15702.49	-20408.23	-21038.66	-20868.16
SBC	Normal	10418.46	8331.39	9012.16	7612.83	5391.76	7150.77	9502.00	6887.32	9059.97	7580.57	9994.46	10331.45	10129.48
	St	10608.40	8573.56	9158.35	7763.28	5587.56	7298.62	9726.59	7066.80	9275.52	7843.36	10194.68	10507.33	10413.03
	HSt	10607.58	8573.02	9157.54	7762.66	5587.77	7298.05	9726.06	7066.36	9274.74	7842.85	10194.18	10506.64	10412.62
	SGT	10605.57	8569.45	9157.21	7774.69	5581.48	7298.08	9726.90	7066.48	9269.11	7836.55	10188.98	10504.14	10418.90
Panel C														
Normal	LR	398.43	500.10	314.12	347.33	402.22	318.07	473.96	381.36	442.12	535.59	413.20	369.60	603.06
	P value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
St	LR	10.486	7.778	13.735	38.552	3.023	14.542	16.721	14.732	3.081	2.146	4.707	9.772	27.879
	P value	0.015	0.051	0.003	0.000	0.388	0.002	0.001	0.002	0.379	0.543	0.195	0.021	0.000
HSt	LR	12.121	8.854	15.361	39.801	2.600	15.693	17.782	15.599	4.647	3.159	5.698	11.153	28.705
	P value	0.007	0.031	0.002	0.000	0.457	0.001	0.000	0.001	0.200	0.368	0.127	0.011	0.000

Table 8. Distributional fitting for the developed stock exchanges for the bear markets: Panel A – distributional parameters for the Normal (N), Student's t (St), Hansen's Skewed t (HSt) and the Skewed Generalised t (SGt) distributions; Panel B – the results of the diagnostic tests for the – Log Likelihood test (-LogL), the Akaike Information Criterion (AIC) and the Schwartz Bayesian Criterion (SBC); Panel C – the results of the Likelihood ratio tests (LR) for the null hypothesis that the simplified version of SGt is preferred.

		Canada	Finland	France	Germany	Greece	Italy	NL	Portugal	Singapore	Sweden	Switzerl	UK	US
Panel A														
N	σ	0.016	0.025	0.018	0.018	0.022	0.018	0.018	0.014	0.014	0.016	0.016	0.016	0.017
St	ν	2.754	3.890	3.779	3.926	3.755	3.695	2.618	4.193	3.340	4.797	3.033	3.329	3.289
	σ	0.018	0.025	0.019	0.019	0.023	0.019	0.022	0.014	0.014	0.017	0.018	0.017	0.018
HSt	ν	2.737	3.888	3.783	3.902	3.754	3.698	2.603	4.137	3.321	4.794	3.020	3.329	3.267
	λ	0.015	0.000	-0.003	0.015	-0.002	0.000	0.013	0.015	0.015	0.015	0.014	0.001	0.013
	σ	0.018	0.025	0.019	0.019	0.023	0.019	0.022	0.014	0.014	0.017	0.018	0.017	0.018
SGT	ν	2.494	5.655	4.048	4.366	4.886	6.089	3.009	5.341	4.661	6.550	3.195	3.298	3.874
	λ	0.021	0.001	0.015	0.014	-0.002	0.015	0.013	0.015	0.015	0.015	0.015	0.013	-0.006
	k	2.239	1.601	1.888	1.839	1.661	1.487	1.733	1.692	1.558	1.712	1.896	2.007	1.741
	σ	0.019	0.025	0.019	0.019	0.022	0.018	0.020	0.014	0.014	0.017	0.017	0.017	0.018
Panel B														
-LogL	Normal	-2904.53	-2989.33	-3460.23	-4416.45	-4578.57	-4757.54	-3127.38	-4860.35	-4139.41	-3025.20	-3049.38	-2934.72	-2813.35
	St	-3062.36	-3071.73	-3550.91	-4528.61	-4725.98	-4884.30	-3281.99	-4970.59	-4271.45	-3069.40	-3165.15	-3032.84	-2920.68
	HSt	-3062.51	-3071.73	-3550.92	-4527.99	-4725.99	-4884.30	-3281.93	-4970.15	-4271.37	-3068.99	-3164.85	-3032.84	-2920.26
	SGt	-3062.64	-3073.13	-3550.55	-4528.28	-4727.27	-4887.17	-3282.39	-4971.24	-4273.11	-3069.63	-3164.84	-3032.68	-2921.12
AIC	Normal	-5805.07	-5974.66	-6916.45	-8828.90	-9153.14	-9511.07	-6250.75	-9716.69	-8274.83	-6046.40	-6094.77	-5865.45	-5622.71
	St	-6118.72	-6137.46	-7095.83	-9051.21	-9445.96	-9762.59	-6557.98	-9935.18	-8536.91	-6132.80	-6324.31	-6059.67	-5835.36
	HSt	-6119.03	-6137.46	-7095.85	-9049.97	-9445.99	-9762.60	-6557.86	-9934.29	-8536.73	-6131.98	-6323.70	-6059.67	-5834.52
	SGt	-6115.28	-6136.26	-7091.11	-9046.56	-9444.54	-9764.33	-6554.78	-9932.47	-8536.21	-6129.26	-6319.67	-6055.36	-5832.24
SBC	Normal	2897.56	2982.15	3453.03	4409.01	4571.02	4750.02	3120.28	4852.90	4132.14	3018.18	3042.36	2927.74	2806.38
	St	3051.90	3060.96	3540.11	4517.44	4714.65	4873.02	3271.35	4959.42	4260.54	3058.86	3154.61	3022.36	2910.22
	HSt	3052.06	3060.96	3540.12	4516.82	4714.66	4873.02	3271.29	4958.98	4260.46	3058.45	3154.31	3022.37	2909.80
	SGT	3045.21	3055.19	3532.55	4509.67	4708.38	4868.37	3264.66	4952.62	4254.92	3052.07	3147.27	3015.23	2903.68
Panel C														
Normal	LR	316.21	167.60	180.66	223.66	297.40	259.26	310.03	221.78	267.38	88.86	230.91	195.91	215.53
	P value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
St	LR	0.561	2.802	-0.718	-0.650	2.582	5.736	0.796	1.298	3.306	0.465	-0.635	-0.315	0.878
	P value	0.905	0.423	1.000	1.000	0.461	0.125	0.850	0.730	0.347	0.926	1.000	1.000	0.831
HSt	LR	0.255	2.802	-0.737	0.587	2.557	5.735	0.920	2.181	3.476	1.281	-0.024	-0.316	1.715
	P value	0.968	0.423	1.000	0.899	0.465	0.125	0.821	0.536	0.324	0.734	1.000	1.000	0.634

Table 9. Distributional fitting for the emerging stock exchanges for the bull markets: Panel A – distributional parameters for the Normal (N), Student's t (St), Hansen's Skewed t (HSt) and the Skewed Generalised t (SGt) distributions; Panel B – the results of the diagnostic tests for the – Log Likelihood test (-LogL), the Akaike Information Criterion (AIC) and the Schwartz Bayesian Criterion (SBC); Panel C – the results of the Likelihood ratio tests (LR) for the null hypothesis that the simplified version of SGt is preferred.

		Argentina	Chile	China	Columbia	India	Peru	Romania	Saudi	Taiwan	Turkey
Panel A											
N	σ	0.022	0.007	0.015	0.014	0.014	0.012	0.015	0.012	0.012	0.021
St	ν	3.434	4.283	2.878	2.869	3.414	3.316	2.645	2.153	3.068	3.164
	σ	0.023	0.007	0.017	0.015	0.014	0.013	0.017	0.022	0.013	0.022
HSt	ν	3.407	4.259	2.868	2.858	3.379	3.331	2.649	2.159	3.050	3.146
	λ	0.013	0.016	0.015	0.014	0.015	0.015	0.007	0.010	0.015	0.013
	σ	0.023	0.007	0.017	0.015	0.014	0.013	0.017	0.021	0.013	0.022
SGT	ν	6.471	3.750	7.792	3.049	3.653	3.800	3.513	3.637	5.363	3.404
	λ	0.011	0.015	0.007	0.010	0.005	0.014	0.016	0.009	0.016	0.008
	k	1.400	2.242	1.177	1.862	1.879	1.803	1.548	1.251	1.395	1.857
	σ	0.022	0.007	0.015	0.014	0.014	0.012	0.015	0.012	0.012	0.022
Panel B											
-LogL	Normal	-6996.59	-10815.13	-5485.18	-6972.01	-8817.12	-6745.65	-8514.34	-9178.31	-7957.11	-7621.49
	St	-7200.64	-11072.47	-5674.69	-7383.52	-9158.18	-6940.54	-8972.39	-9744.24	-8186.90	-8017.59
	HSt	-7200.37	-11071.80	-5675.18	-7383.73	-9157.91	-6940.10	-8972.52	-9744.73	-8186.85	-8017.07
	SGt	-7208.58	-11072.61	-5686.93	-7384.01	-9158.50	-6940.63	-8976.33	-9757.00	-8192.81	-8017.80
AIC	Normal	-13989.17	-21626.26	-10966.36	-13940.02	-17630.25	-13487.30	-17024.68	-18352.62	-15910.22	-15238.98
	St	-14395.27	-22138.93	-11343.38	-14761.04	-18310.36	-13875.08	-17938.78	-19482.48	-16367.80	-16029.18
	HSt	-14394.74	-22137.60	-11344.36	-14761.46	-18309.83	-13874.20	-17939.04	-19483.47	-16367.70	-16028.14
	SGt	-14407.16	-22135.22	-11363.87	-14758.03	-18306.99	-13871.26	-17942.65	-19504.01	-16375.62	-16025.61
SBC	Normal	6988.62	10807.11	5477.59	6964.21	8809.09	6737.94	8506.31	9170.30	7949.22	7613.44
	St	7188.68	11060.43	5663.30	7371.82	9146.12	6928.97	8960.35	9732.23	8175.08	8005.52
	HSt	7188.42	11059.76	5663.79	7372.04	9145.86	6928.53	8960.48	9732.72	8175.02	8004.99
	SGT	7188.66	11052.54	5667.95	7364.52	9138.41	6921.34	8956.27	9736.98	8173.10	7997.68
Panel C											
Normal	LR	423.99	514.96	403.51	824.01	682.75	389.96	923.98	1157.38	471.41	792.63
	P value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
St	LR	15.891	0.289	24.490	0.991	0.637	0.181	7.873	25.523	11.822	0.426
	P value	0.001	0.962	0.000	0.804	0.888	0.981	0.049	0.000	0.008	0.935
HSt	LR	16.426	1.618	23.508	0.567	1.165	1.062	7.613	24.541	11.924	1.469
	P value	0.001	0.655	0.000	0.904	0.761	0.786	0.055	0.000	0.008	0.689

Table 10. Distributional fitting for the emerging stock exchanges for the bear markets: Panel A – distributional parameters for the Normal (N), Student's t (St), Hansen's Skewed t (HSt) and the Skewed Generalised t (SGt) distributions; Panel B – the results of the diagnostic tests for the – Log Likelihood test (-LogL), the Akaike Information Criterion (AIC) and the Schwartz Bayesian Criterion (SBC); Panel C – the results of the Likelihood ratio tests (LR) for the null hypothesis that the simplified version of SGt is preferred.

		Argentina	Chile	China	Columbia	India	Peru	Romania	Saudi Arabia	Taiwan	Turkey
Panel A											
N	σ	0.022	0.009	0.017	0.012	0.018	0.016	0.018	0.022	0.017	0.023
St	ν	3.022	2.794	2.924	3.620	3.729	2.189	2.311	2.029	4.078	5.370
	σ	0.024	0.010	0.019	0.012	0.019	0.026	0.026	0.090	0.018	0.023
HSt	ν	3.024	2.782	2.912	3.592	3.675	2.206	2.298	2.037	4.053	5.344
	λ	-0.003	0.015	0.015	0.015	0.014	0.008	0.012	0.033	0.015	0.015
	σ	0.024	0.010	0.019	0.012	0.019	0.025	0.027	0.080	0.018	0.023
SGT	ν	4.273	3.379	6.037	7.067	19.274	2.653	3.221	3.579	77.571	7.194
	λ	0.005	0.016	0.002	0.014	-0.003	0.008	0.012	0.033	0.018	0.015
	k	1.533	1.675	1.282	1.403	1.166	1.589	1.461	1.177	1.129	1.749
	σ	0.022	0.009	0.017	0.012	0.018	0.018	0.020	0.024	0.017	0.023
Panel B											
-LogL	Normal	-3086.54	-3936.99	-5613.99	-4105.38	-2903.67	-5479.76	-3108.35	-2822.18	-4115.90	-2645.70
	St	-3211.37	-4085.35	-5820.66	-4191.46	-2963.98	-5942.42	-3304.28	-3041.34	-4175.53	-2679.34
	HSt	-3211.38	-4085.36	-5820.34	-4191.11	-2963.64	-5942.80	-3304.34	-3042.34	-4174.90	-2678.96
	SGt	-3212.83	-4086.01	-5829.05	-4194.74	-2969.90	-5944.63	-3306.69	-3049.01	-4188.41	-2679.29
AIC	Normal	-6169.07	-7869.98	-11223.98	-8206.76	-5803.34	-10955.52	-6212.70	-5640.35	-8227.80	-5287.40
	St	-6416.74	-8164.71	-11635.33	-8376.93	-5921.96	-11878.83	-6602.57	-6076.67	-8345.05	-5352.68
	HSt	-6416.76	-8164.71	-11634.68	-8376.21	-5921.29	-11879.60	-6602.67	-6078.67	-8343.79	-5351.93
	SGt	-6415.66	-8162.02	-11648.09	-8379.47	-5929.80	-11879.26	-6603.38	-6088.02	-8366.81	-5348.59
SBC	Normal	3079.38	3929.92	5606.33	4098.17	2896.65	5472.16	3101.26	2815.10	4108.56	2638.67
	St	3200.63	4074.74	5809.17	4180.65	2953.44	5931.01	3293.65	3030.72	4164.51	2668.80
	HSt	3200.64	4074.75	5808.85	4180.30	2953.11	5931.40	3293.70	3031.72	4163.88	2668.42
	SGT	3194.94	4068.33	5809.89	4176.72	2952.34	5925.62	3288.97	3031.32	4170.05	2661.73
Panel C											
Normal	LR	252.59	298.03	430.11	178.72	132.46	929.73	396.68	453.67	145.01	67.19
	P value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
St	LR	2.926	1.309	16.764	6.546	11.844	4.422	4.814	15.348	25.758	-0.093
	P value	0.403	0.727	0.001	0.088	0.008	0.219	0.186	0.002	0.000	1.000
HSt	LR	2.903	1.302	17.412	7.261	12.513	3.656	4.713	13.346	27.018	0.661
	P value	0.407	0.729	0.001	0.064	0.006	0.301	0.194	0.004	0.000	0.882

Figure 1 Daily movement of the S&P500 stock market index for the US for the period 1 January 2000 and 31 January 2016 (blue line) and its separation into the bull and bear sub-periods (orange line). The high (low) values of the orange line indicate the bull (bear) periods.

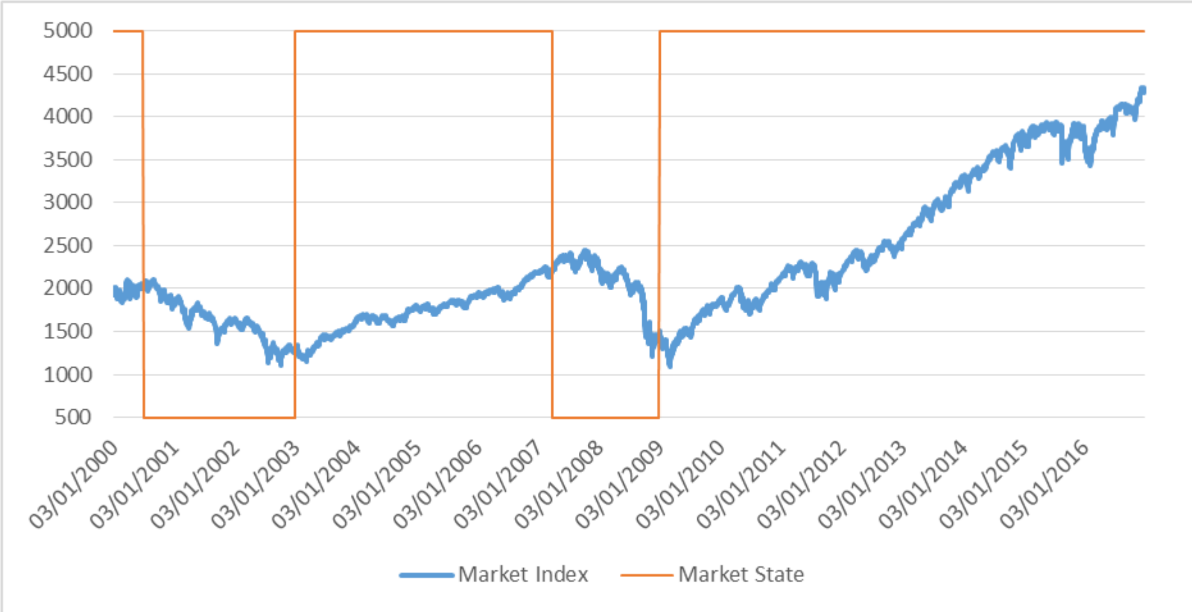
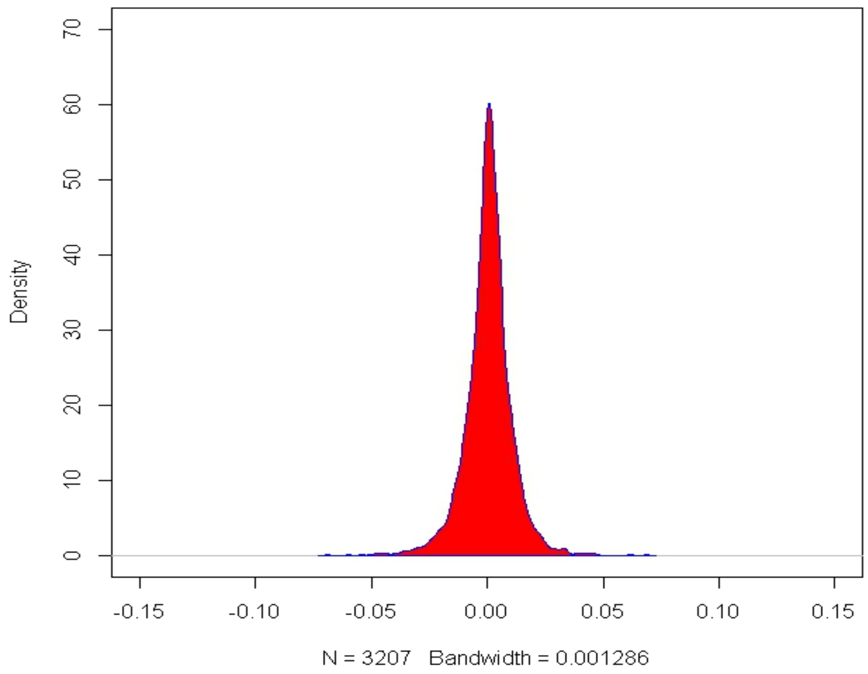


Figure 2. Daily movement of the BIST National 100 stock market index for Turkey for the period 1 January 2000 - 31 January 2016 (blue line) and its separation into the bull and bear sub-periods (orange line). The high (low) values of the orange line indicate the bull (bear) periods.

Figure 3. Kernel density for daily returns of the S&P 500 index separated for the bull markets (Panel A) and the bear markets (Panel B) for the period of 1 January 2000 – 31 December 2016.

Panel A



Panel B

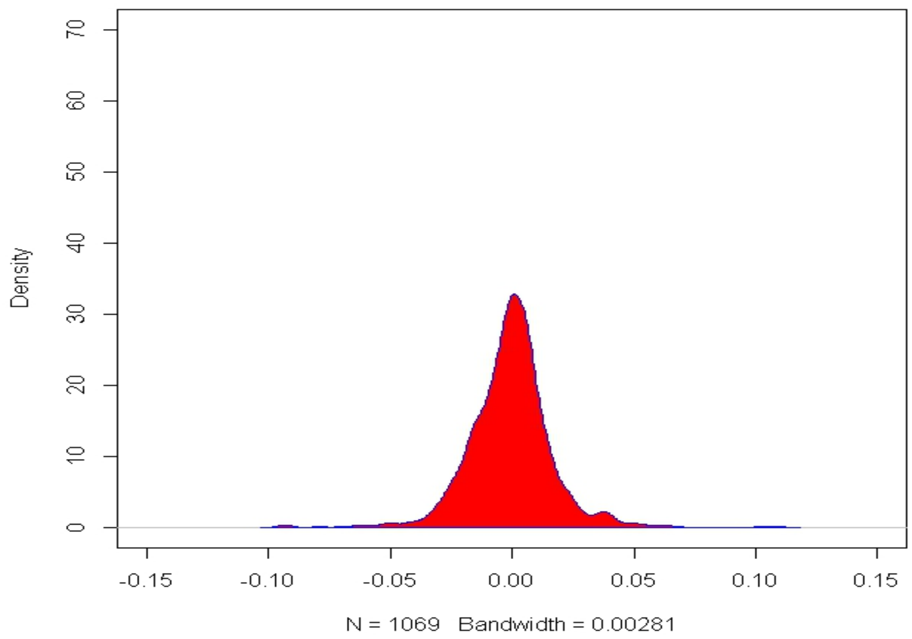
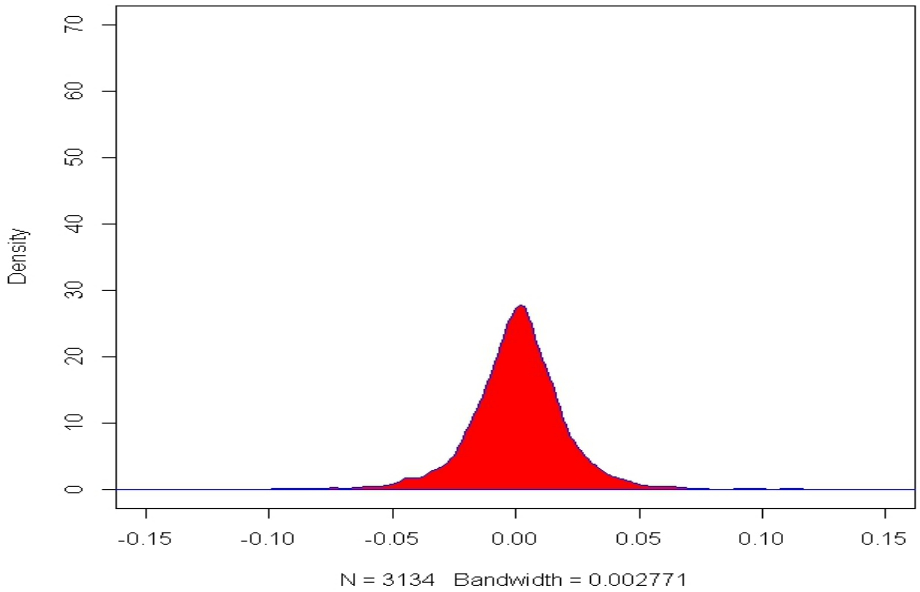


Figure 4. Kernel density for daily returns of the BIST National 100 stock market index for Turkey separated for the bull markets (Panel A) and the bear markets (Panel B) for the period of 1 January 2000 – 31 December 2016.

Panel A



Panel B

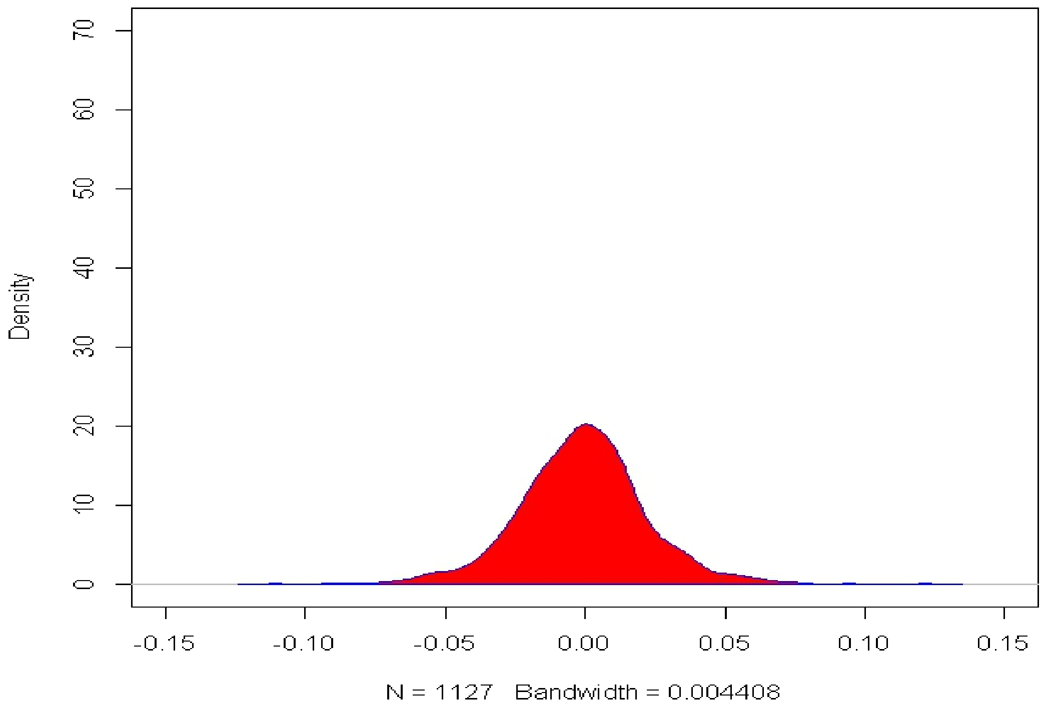
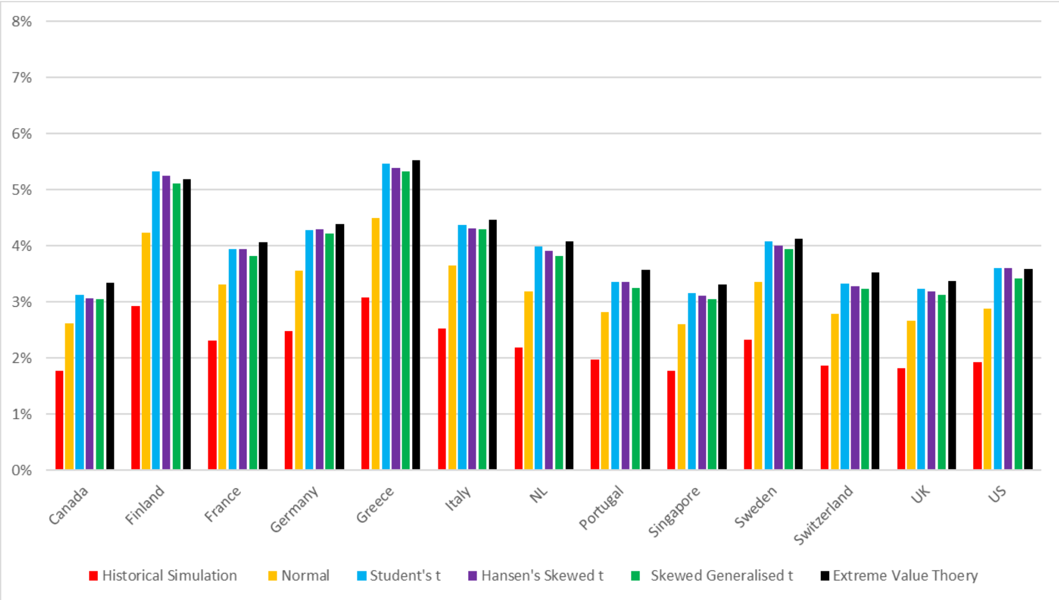


Figure 5. VaR estimates for the developed stock exchanges (Panel A) and the emerging stock exchanges (Panel B).

Panel A



Panel B

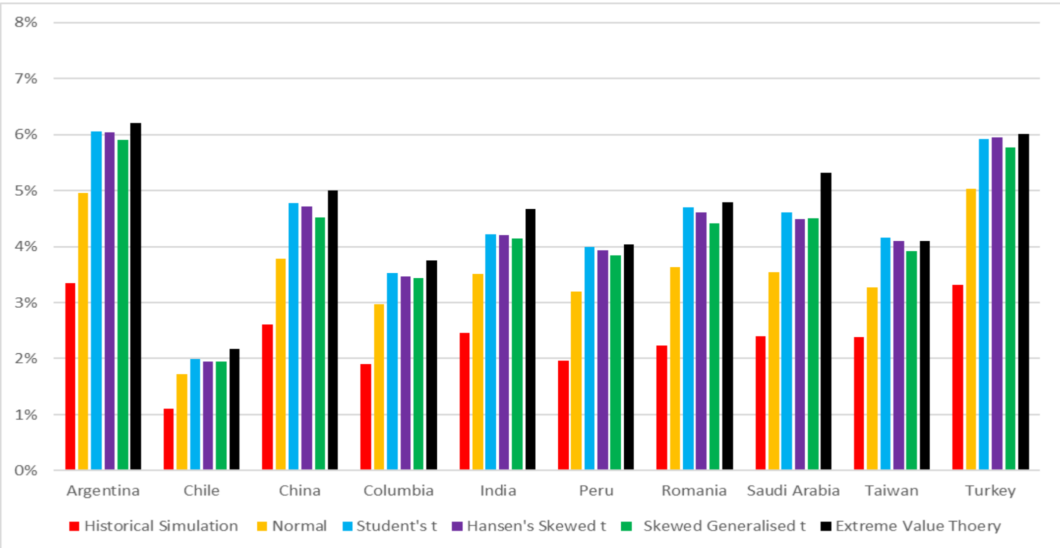
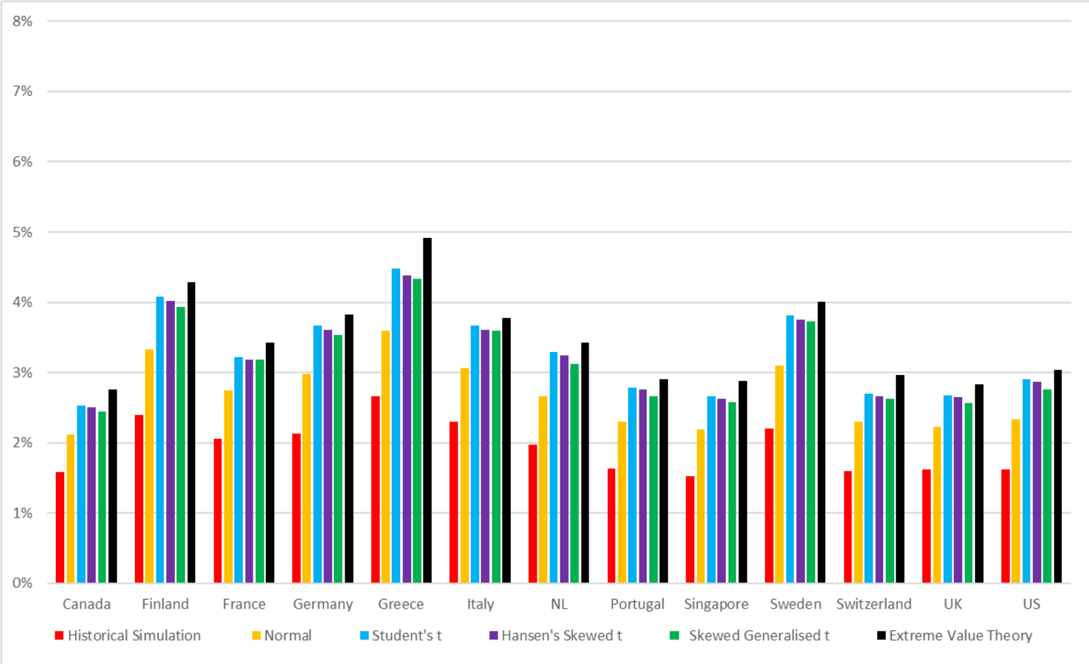


Figure 6. VaR for the developed stock exchanges for the bull markets (Panel A) and bear markets (Panel B)

Panel A



Panel B

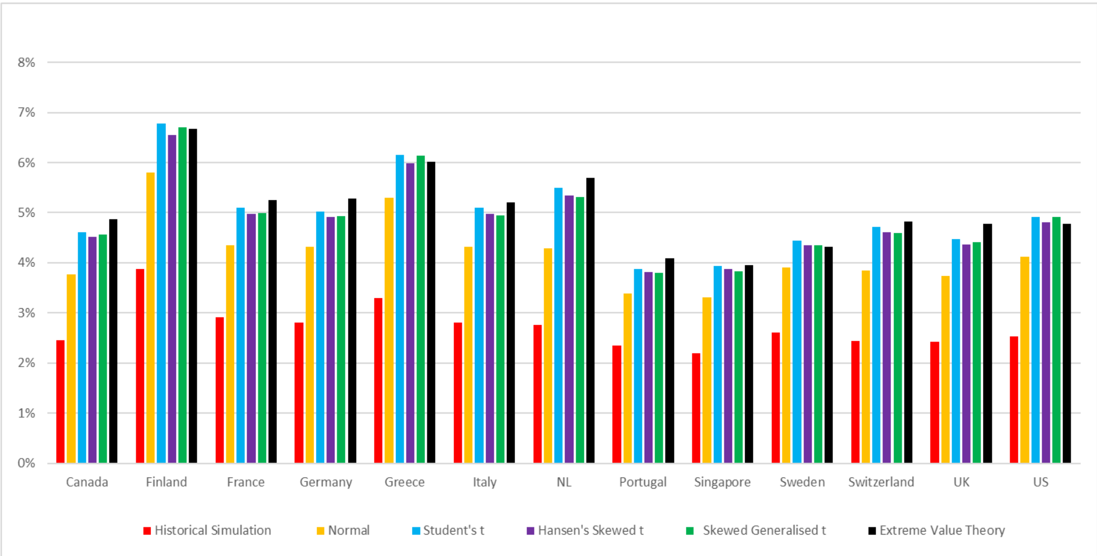
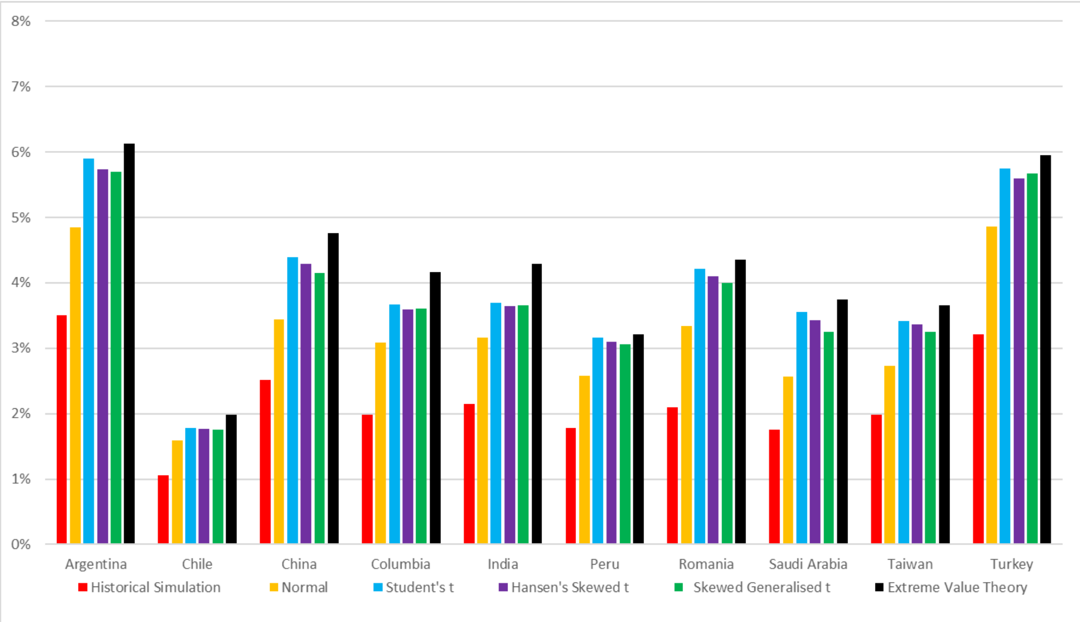


Figure 7. VaR for the emerging stock exchanges for the bull markets (Panel A) and bear markets (Panel B)

Panel A



Panel B

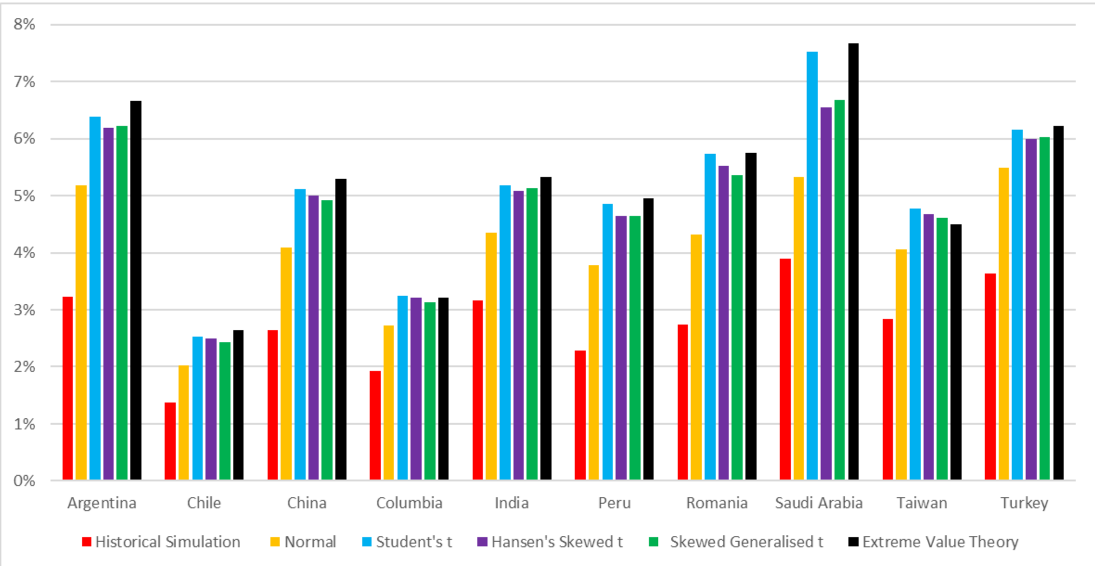
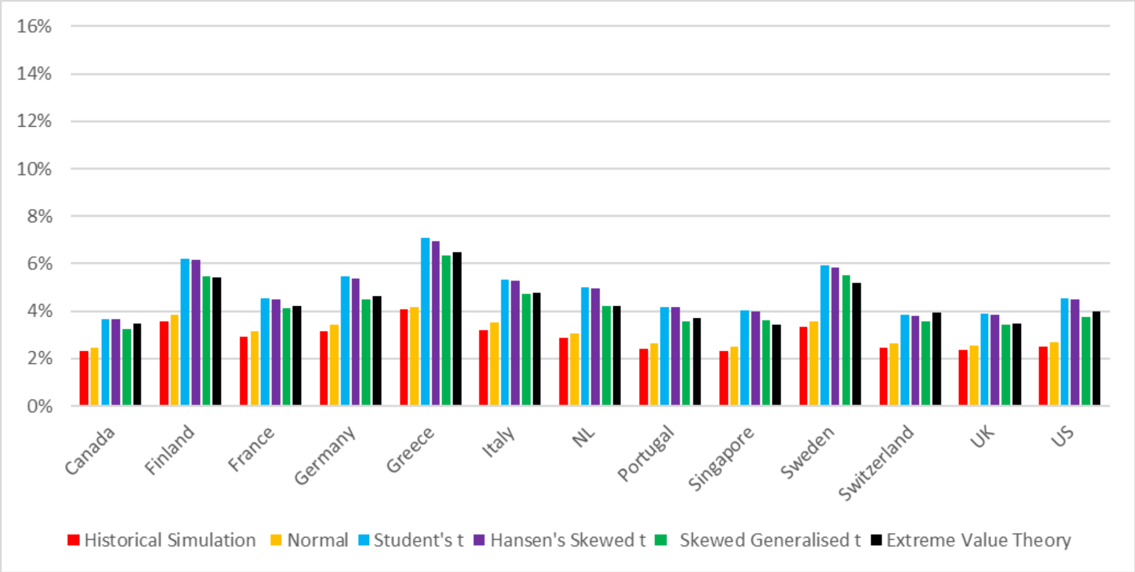


Figure 8. CVaR for the developed stock exchanges for the bull markets (Panel A) and bear markets (Panel B)

Panel A



Panel B

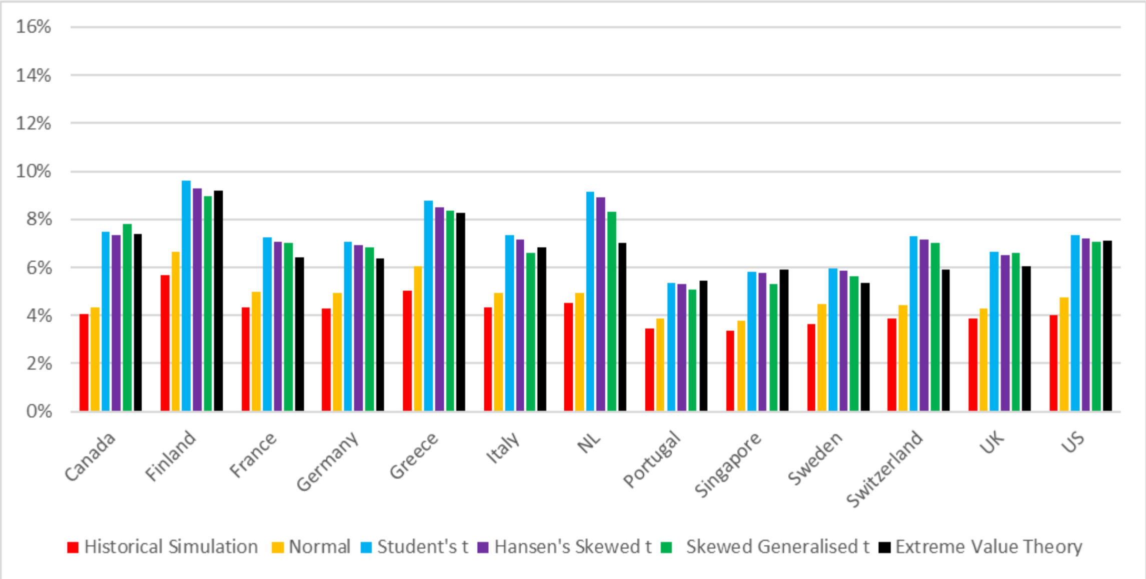
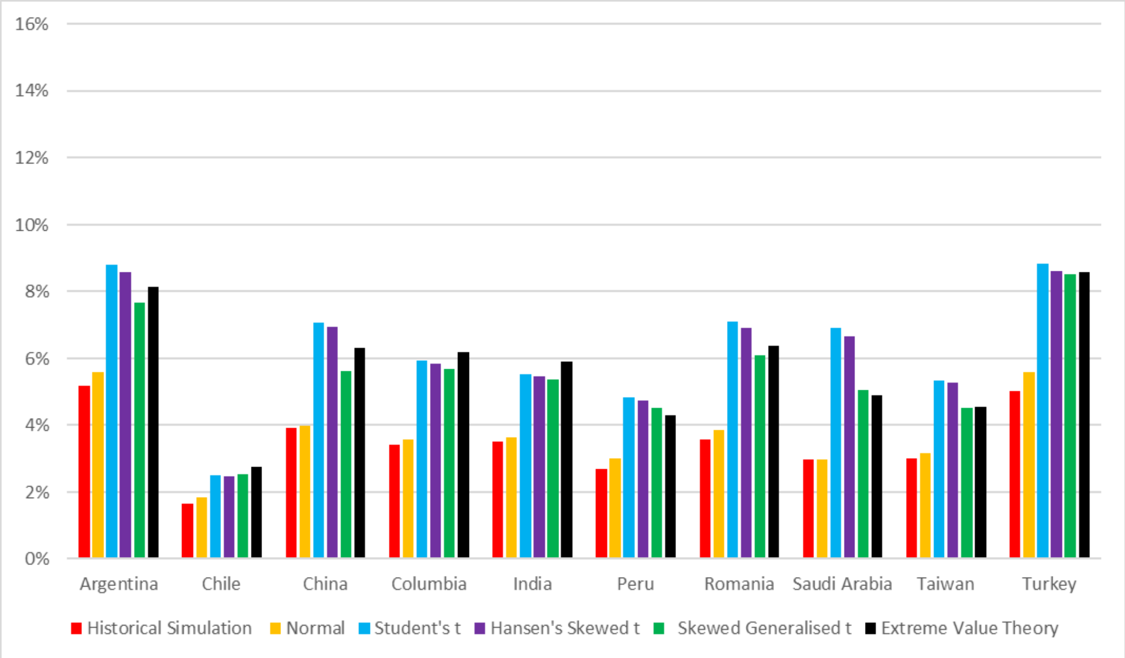


Figure 9. CVaR for the emerging stock exchanges for the bull markets (Panel A) and bear markets (Panel B)

Panel A



Panel B